**Dark Matter in Galaxy Clusters:**

Estimating the Virial mass of a cluster:

Consider a set of $n$ galaxies, each of mass $m$. Let $\overline{V^2}$ be the measured time averaged squared velocity of a galaxy and $\sigma^2$ the average of this quantity over the $n$ galaxies.

$\sigma$ represents the "velocity dispersion" which is the root-mean-squared velocity of the cluster galaxies about the mean velocity (typically ~1000 km/s).

Then the time averaged KE of the system is:

$$\overline{K} = n\frac{1}{2}m\sigma^2 = \frac{1}{2}M\sigma^2 \quad (M = nm)$$

The gravitational potential for two galaxies separated by a distance $r$ is then $-\frac{Gm^2}{r}$

Let $\frac{1}{r_{cl}}$ be the cluster average of the time average of $\frac{1}{r}$. There are $\frac{n(n-1)}{2}$ pairs of galaxies so the time averaged potential of the system is then $\overline{V} = -\frac{n(n-1)}{2}\frac{Gm^2}{r_{cl}}$

According to the Virial Theorem:

$$\overline{K} = -\frac{1}{2}\overline{V}$$

Note that the number of galaxies in a cluster is typically >> 1, so that:

$$n(n-1) \cong n^2$$

$$\frac{1}{2}M\sigma^2 = \frac{1}{2}\frac{n(n-1)}{2}\frac{Gm^2}{r_{cl}} = \frac{1}{2}\frac{GM^2}{2r_{cl}}$$

$$M = \frac{2r_{cl}\sigma^2}{G}$$

or order of magnitude:

$$\boxed{M \sim \frac{r_{cl}\sigma^2}{G}}$$

**Virial mass, gravitational "lensing" mass, & intergalactic medium mass determinations verses luminosity mass of clusters:**

Virial mass estimate:

$$M \sim \frac{r_{cl}\,\sigma^2}{G}$$

$\sigma \sim 1000\ \frac{km}{s} = 10^6\ \frac{m}{s}$

$r_{cl} \sim 1\ Mpc = 3.1 \times 10^{22}\ m$

$G = 6.7 \times 10^{-11}\ \frac{N m^2}{kg^2}$

$$M \sim \frac{3.1 \times 10^{22} \times (10^6)^2}{6.7 \times 10^{-11}}\ kg$$

$$M = 4.6 \times 10^{44}\ kg \cong 2 \times 10^{14}\ M_\odot$$

Gravitational "lensing" mass estimate:

$D_{ol} \equiv$ dist to "lense" $\cong 1\ Gpc$

$\theta_E \equiv$ Einstein angle $\cong 0.5$ arcmin

$$M \sim \frac{\theta_E^2\, c^2\, D_{ol}}{4G}$$

$$M = 4 \times 10^{44}\ kg \cong 2 \times 10^{14}\ M_\odot$$

Intergalactic medium (cluster) mass estimate: (ie.: mass of the whole cluster, not just the gas)

$$M \sim 2 \times 10^{14}\ M_\odot$$

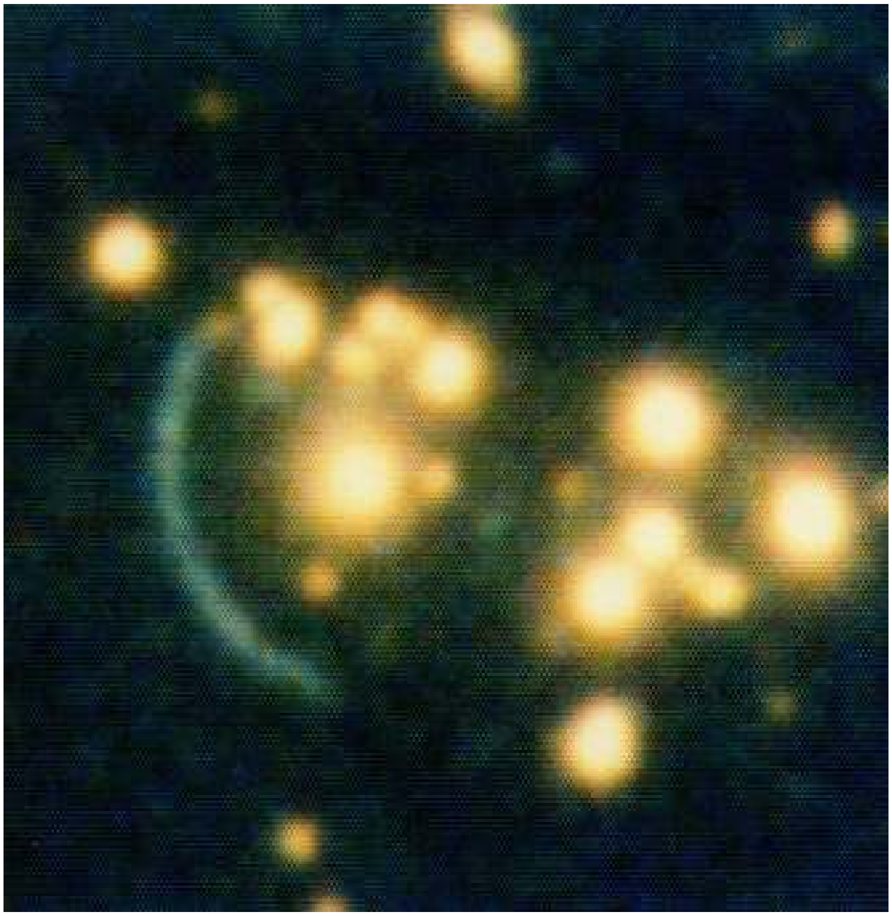
Luminosity (cluster) mass estimate (includes IGM gas):

$L_* \approx 2 \times 10^{10}\ L_\odot \rightarrow 2 \times 10^{11}\ M_\odot$

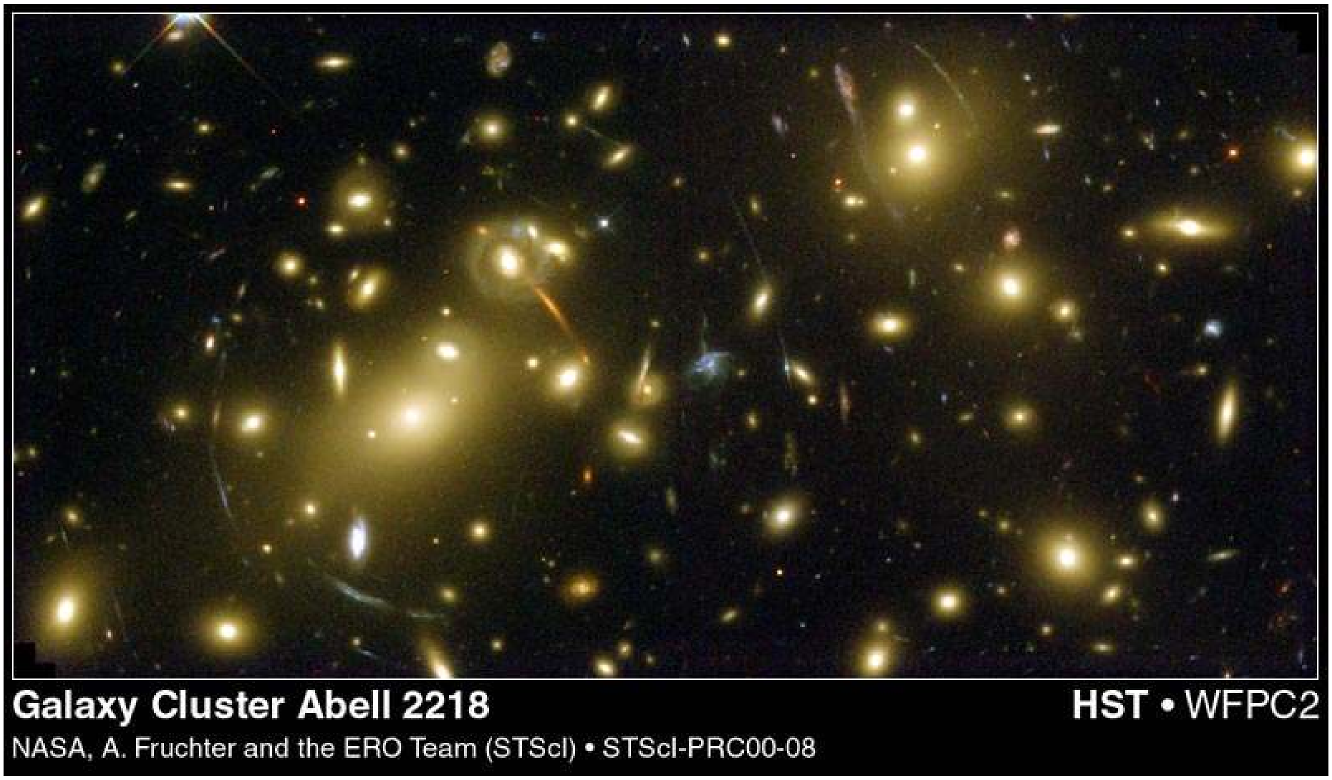
$L_{cl} \approx 100\ L_* \rightarrow 2 \times 10^{13}\ M_\odot$

$$M \sim 3 \times 10^{13}\ M_\odot$$

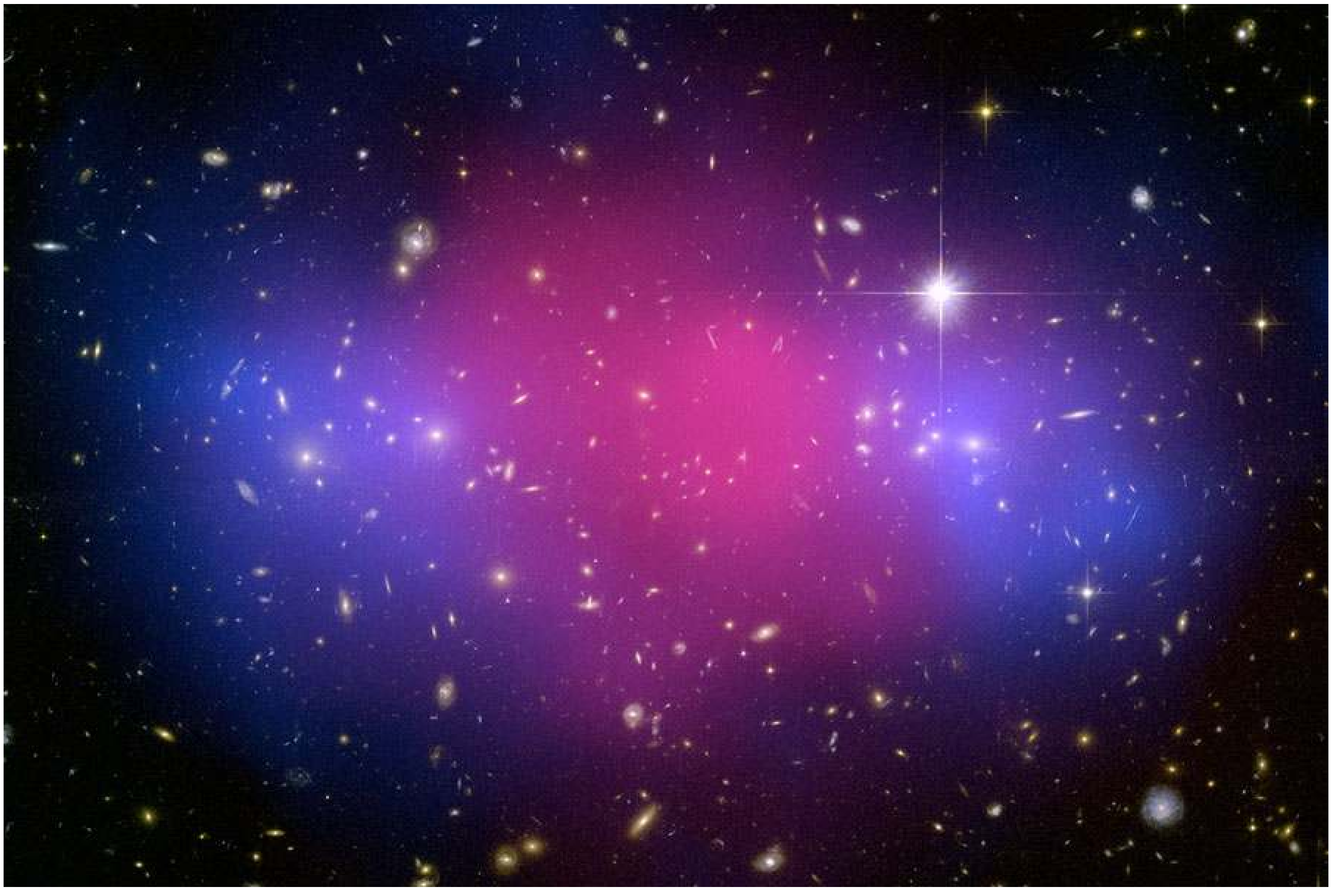
Only ~15% of the total cluster mass is made up of normal stars and gas!!!



This giant blue arc in galaxy cluster CL2244-02 is actually a distorted image of a blue background galaxy about twice as far away as the cluster. The cluster gravitational field acts like a giant lens producing a distorted, magnified image of the background galaxy. Graviational lens images like this can be used to trace the amount and distribution of dark matter in galaxy clusters. All the yellowish red objects are galaxies in the foreground galaxy cluster CL2244-02. The blue arc and the small blue elongated dot in the middle of the image are highly distorted images of a blue background galaxy. Three-color image processed by Dr. Joseph H. Jones at CFHT, 1988.




**Another spectacular example of gravitational lensing through a foreground galaxy cluster, imaged by the HST. Note the many galaxy "arcs" (gravitational images of background galaxies by the gravity field of the foreground galaxy cluster). These "arcs" can be used to map out the dark matter distribution in a galaxy cluster. Generally the dark matter is distributed in a similar manner to the luminous (normal baryonic) matter.**



**Finally, below is a HST image of two galaxy clusters caught in the act of colliding (having passed through one another) and in the process partially separating the dark matter and the normal matter. Notice the many galaxy "arcs" surrounding the two galaxy clusters on either side of the image. These have been used to map the dark matter contained in the two clusters. The map of the dark matter is depicted in this image with the diffuse purplish hue. Since dark matter only interacts with itself and normal matter through gravity, when the two galaxy clusters passed through each other the clouds of dark matter (and most of the individual galaxies) simply passed by each other with no effect other than some gravitational perturbations. However, the vast clouds of extremely hot but diffuse hydrogen gas that are known to fill the space between galaxies in galaxy clusters, did "collide" together halting their forward motions as the galaxy clusters passed through each other. The X-ray emission from this hot cloud is shown in pink (imaged by the Chandra X-ray Telescope) as it has been separated out from the visible galaxies and the dark matter continuing away from each other on either side of the image.**