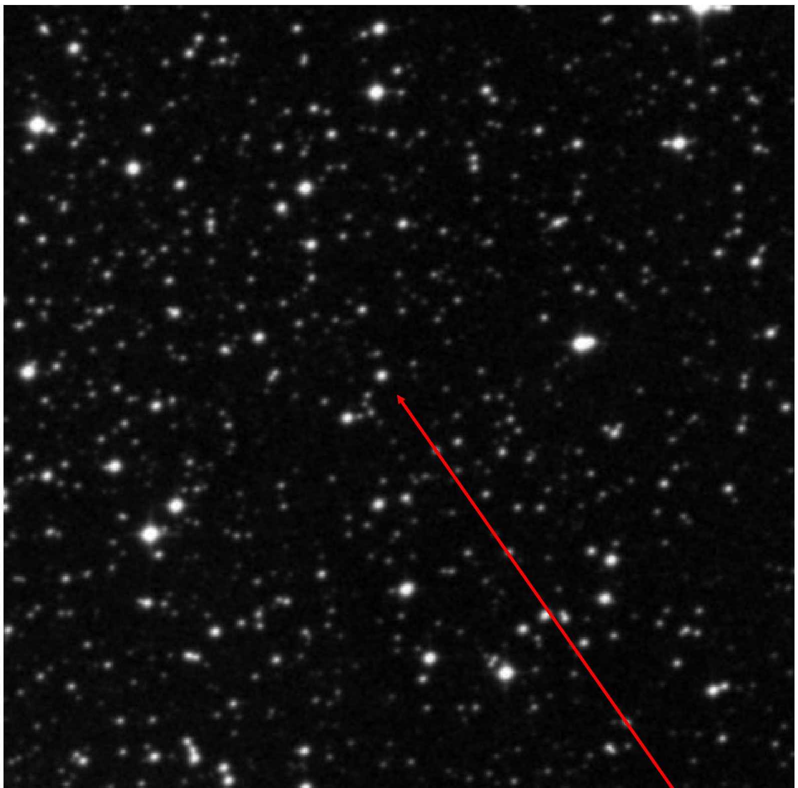
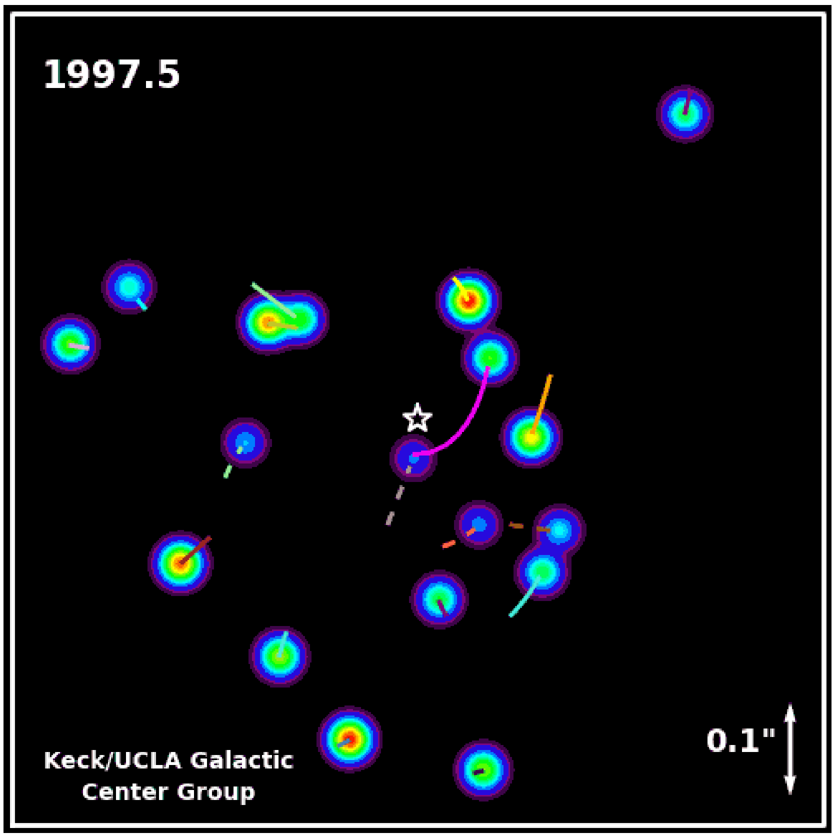


Sagittarius A*

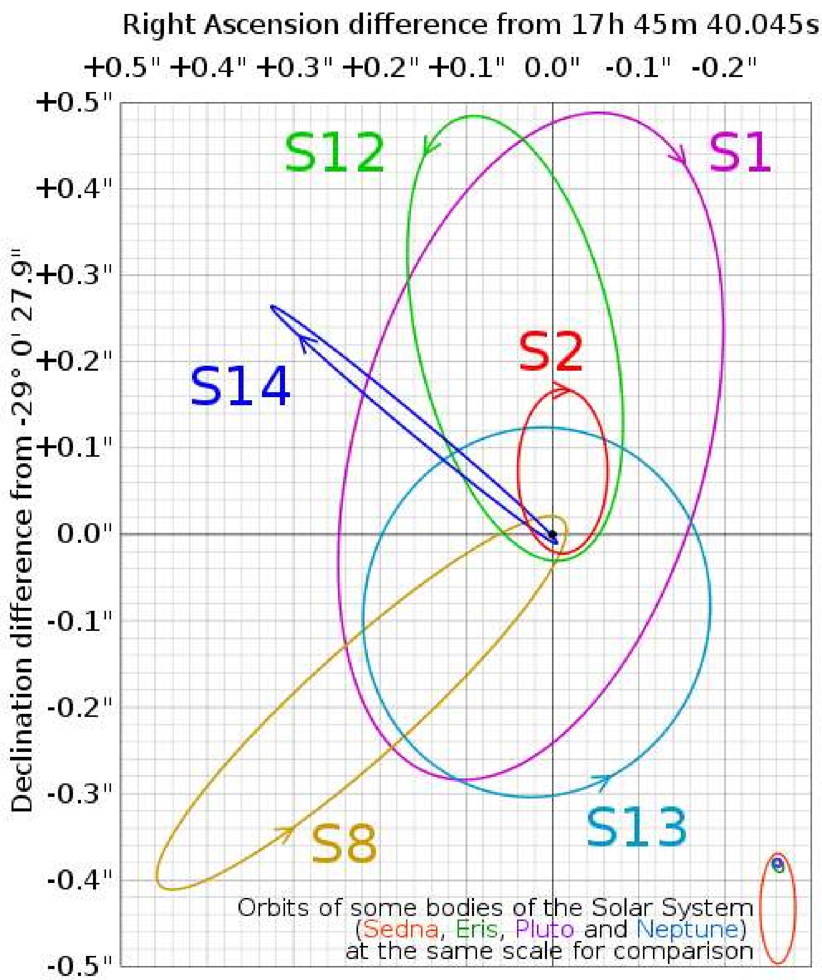
8' x 8' visual image of the exact center of the Milky Way

The actual center is blocked by dust and is not visible. At the distance to the center (26,000 ly), this image would span 60 ly. This is the FOV of NGAO's CCD camera. Our camera currently reads out 512 x 512 pixels, and each pixel is approximately 1" x 1" square, about 1/3 the diameters of the star images seen above. The adaptive optics movie on the next page spans only **one pixel** in the above image!



1" x 1" IR adaptive optics time-lapse movie image of the exact center of the Milky Way.

This image spans only 0.126 ly (1/8 ly) or 7500 AU. Notice that star "S0-2" completes an entire orbit. Using this star especially, but also the other stellar orbits, the mass of the object that it is orbiting may be accurately determined using the techniques of visual binary star orbit analysis.



From the orbital analysis of S0-2 (S2 on figure):

P = 15.2 y

a = 982 AU

r_p = 120 AU

using special astronomical units: a (AU), P (years), m_1 & m_2 in solar mass units ($M_\odot$)

Kepler's 3rd law becomes:

$$m_1 + m_2 = \frac{a^3}{P^2}$$

And for this situation, since

$$m_1 \gg m_2$$

we can write:

$$m_1 = \frac{a^3}{P^2}$$

Where m_1 = Sag A* mass

$$M_{SagA^*} = \frac{(982\,AU)^3}{(15.2\,y)^2} = 4.1 \times 10^6 M_\odot$$

Now to evaluate whether or not Sag A* is a black hole, we must introduce some concepts of General Relativity.

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Principle of Equivalence:

There is no local experiment that can be done to distinguish between the effect of a uniform gravitational field in a non-accelerating inertial frame and the effects of a uniformly accelerating (non-inertial) reference frame.

In Special Relativity, physics is the same for an observer in any inertial frame, no matter what the velocity of that frame. In General Relativity, Einstein says that physics must be the same in any reference frame, even accelerated (non-inertial) ones. As in Special Relativity this connects time to the 3 spacial coordinates such that events are described as taking place in a 4 dimensional "space-time". The geometry of space-time is how the effects of gravity is described using General Relativity.

Space-time interval:

This is the (infinitesimal) "separation between two events (such as a particle passing through successive points) in space-time". The trajectory of a particle in space-time can be regarded as a collection of infinitesimal intervals. Since the particle is following the contours of space-time, the interval serves both to define the trajectory and to represent the shape of space-time.

Below is a generalized space-time interval for curved space-time. The four coefficients describe the curvature of the space-time and its deviation from a Euclidian nature (flat space-time). All the coefficients would be equal to 1 in flat space-time.

$$(ds)^2 = g_0 (c\,dt)^2 - g_1 (dx)^2 - g_2 (dy)^2 - g_3 (dz)^2$$

The figures below depict "3 dimensional space-times" of different "curvatures". (Since we can't really visualize 4 dimensional space-times.)

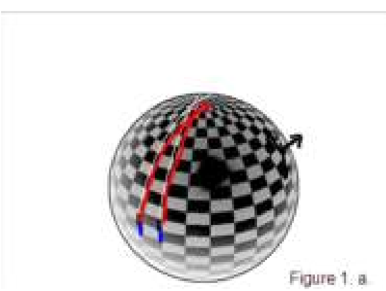

Figure 1. a

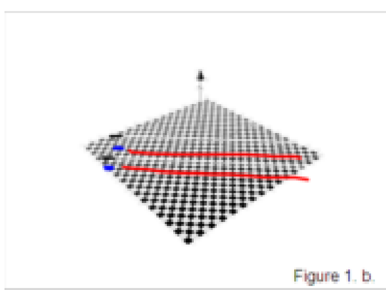

Figure 1. b

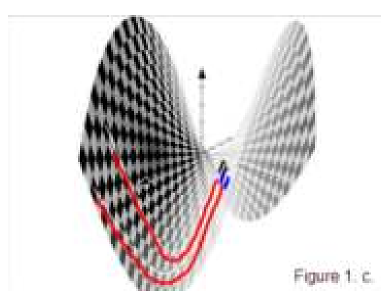

Figure 1. c

General Relativity gives a relationship between the curvature of space-time and the mass-energy density in space-time:

curvature of space = $8\pi G/c^4$ (mass-energy density)

Where both the "curvature of space" (the coefficients of the space-time interval) and the "mass-energy density" are represented mathematically by tensors. "Solving the equations" generally means to determine the curvature of space time given the mass-energy density in a given region of space-time. In GR, the effects of gravity are determined by the way a mass or energy density curves space-time. It is tempting to think about using $E = mc^2$ and just using classical gravity on the equivalent mass of some energy density, but this does not give the correct answer. The principle of equivalence leads to depicting the gravity as the geometry of space-time, and it is this geometry that gives the correct description of the effects of gravity.

Is Sag A* likely a black-hole?

Within a year of the 1916 publication of General Relativity, Karl Schwarzschild worked out the solutions to the equations for the curvature of space-time near a spherically symmetric mass M.

The space-time interval for this solution in spherical coordinates is:

$$(ds)^2 = c^2\left(1 - \frac{2GM}{c^2 r}\right)(dt)^2 - \frac{(dr)^2}{\left(1 - \frac{2GM}{c^2 r}\right)} - r^2(d\theta)^2 - r^2\sin^2\theta(d\phi)^2$$

Note that the radial part of the solution can "blow up" at a particular radius r_S (Schwarzschild radius):

$$r_s = \frac{2GM}{c^2} \quad \left(\text{note: } V_{esc} = \sqrt{\frac{2GM}{r}} \rightarrow V_{esc} = c \rightarrow r = \frac{2GM}{c^2}\right)$$

This is also the radius at which the "escape velocity" for a spherical mass becomes the speed of light. Nothing can escape once it passes beyond this point and any spherical mass distribution that has a radius smaller than the Schwarzschild radius for its mass is defined to be a "black hole".

Let us analyze the data from the observations of Sag A* to determine the likelihood that it is a black hole.

$$M_{SagA^*} = 4.1\times10^6 M_\odot \qquad r_s = \frac{2GM}{c^2}$$

$$= \frac{2\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)\left(4.1\times10^6 M_\odot\right)\left(2\times10^{30}\frac{kg}{M_\odot}\right)}{\left(3\times10^8\frac{m}{s}\right)^2}$$

$$= 1.22\times10^{10} m \qquad 1AU = 1.5\times10^{11} m$$

$$= 0.08\ AU$$

From the orbit of S0-2, the closest approach is 120 AU. However other observations constrain the radius to less than 45 AU. Any mass distribution with 4.1 million solar masses in a spherical volume of 45 AU radius would have collapsed into a black hole on a time scale much less than the current age of the Milky Way Galaxy. This is the best empirical evidence for the existence of any black hole, super-massive or otherwise.

Note: the radio source Sag A* is not actually the black hole itself, but is thought to be material very near the black hole perhaps within an accretion disk around the super-massive black hole.