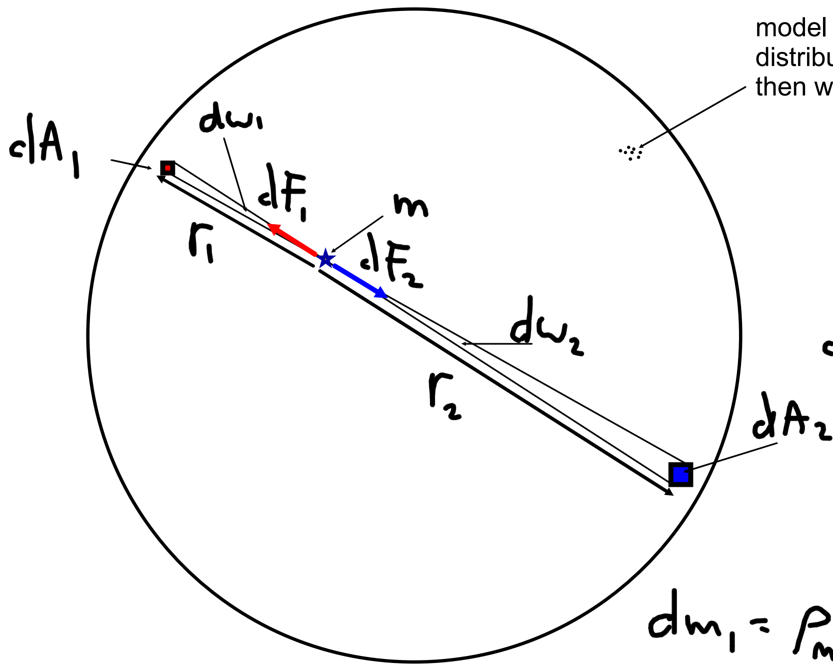
Gravitational effect of an "external" spherical mass distribution on a concentric spherical region at the center of the spherical mass distribution:

Case of a very thin spherical shell on a mass point inside the shell:



model shell with uniformly distributed equal mass points, then we can write:

$$\rho_{mp} = \frac{m_p N_p}{A_p} = \text{const}$$

Using symmetry arguments:

$\overrightarrow{dF_1}$ opposite direction of $\overrightarrow{dF_2}$

$$dA_1 = r_1^2 dw_1^2$$
$$dA_2 = r_2^2 dw_2^2$$

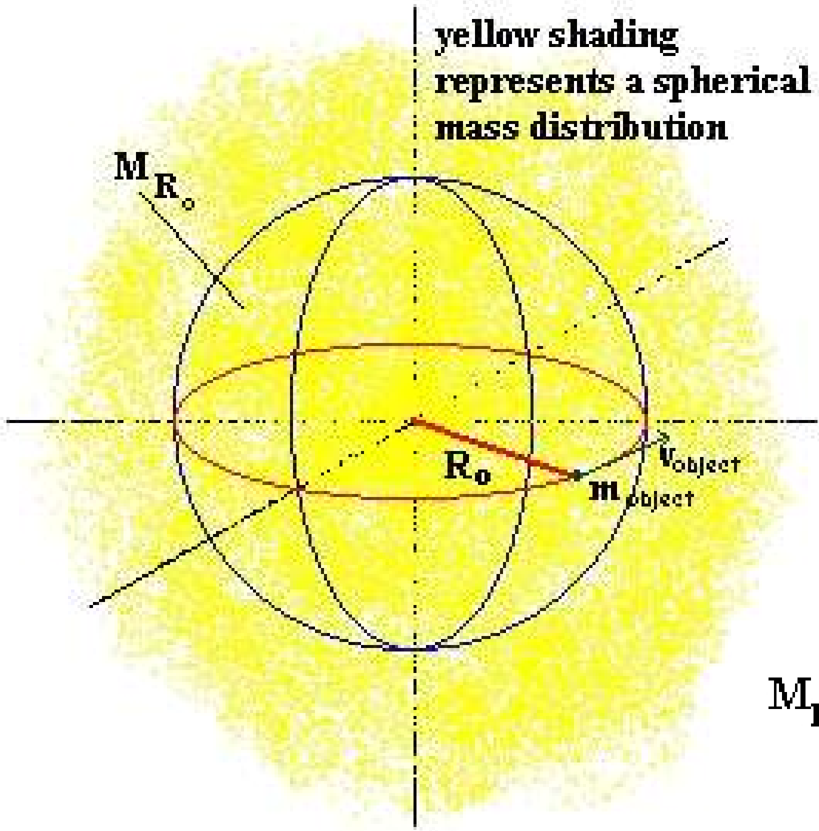
$$dm_1 = \rho_{mp} dA_1 = \rho_{mp} r_1^2 dw_1^2$$
$$dm_2 = \rho_{mp} dA_2 = \rho_{mp} r_2^2 dw_2^2$$

$$dF_1 = \frac{Gm\,dm_1}{r_1^2} = \frac{Gm\rho_{mp} r_1^2 dw_1^2}{r_1^2} \qquad dF_2 = \frac{Gm\,dm_2}{r_2^2} = \frac{Gm\rho_{mp} r_2^2 dw^2}{r_2^2}$$

$$dF_{NET} = dF_1 - dF_2 = Gm\rho_{mp}\left[\frac{r_1^2}{r_1^2}dw_1^2 - \frac{r_2^2}{r_2^2}dw_2^2\right]$$

Note: $dw_1 = dw_2$

So: $dF_{NET} = 0$ and an integral over all angles would also yield zero force; furthermore, a thick shell is modeled by many concentric thin shells and the **<u>net force</u>** would still be **<u>zero</u>**



$$M_{R_\circ} = \frac{R_o \times v_{object}^2}{G}$$

Using F = ma for a circular orbit:

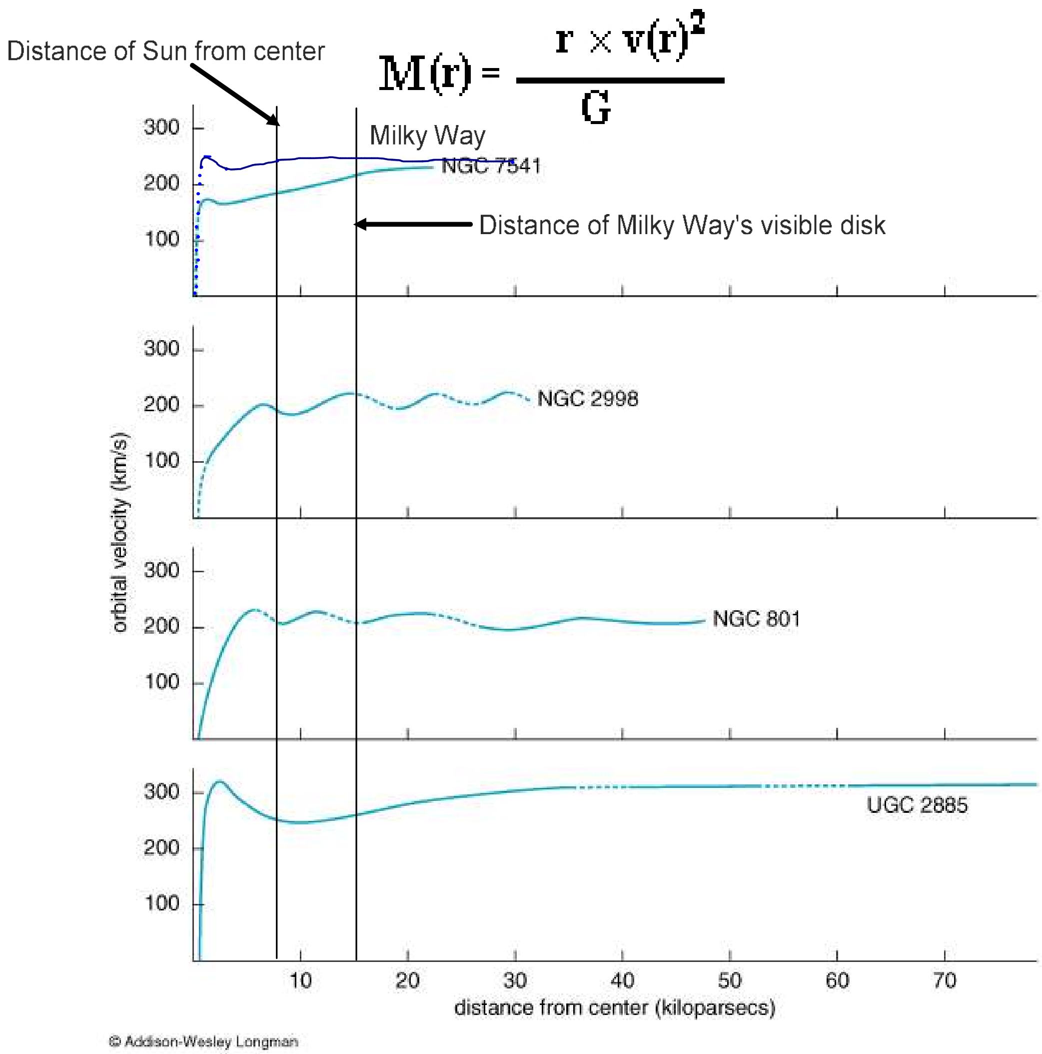
$$m_{object}\, a = \frac{G M_{R_o} m_{object}}{R_o^2}$$

Definition of centripetal acceleration:

$$a = a_c = \frac{v_{object}^2}{R_o}$$

$$\frac{v_{object}^2}{R_o} = \frac{G M_{R_o}}{R_o^2} \longrightarrow M_{R_o} = \frac{R_o v_{object}^2}{G}$$

$$M(r) = \frac{r \times v(r)^2}{G}$$



$$V(r) = \text{const} \longrightarrow M(r) = \left(\frac{\text{const}^2}{G}\right) r = C\,r$$

Note that the "flat" rotation curves for most disk galaxies imply much more mass than the mass contained in the visible stars of the galaxies.