Exercise 5: Wien's & Stefan-Boltzmann laws from Planck's equation

Planck's equation:

$$R(\lambda) = \left(\frac{C}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\left(\frac{hc}{\lambda}\right)\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right]$$

a. Differentiate Planck's formula and set equal to zero to derive Wien's law. Note that the solution will involve a non-analytical step (i.e.: a step which involves a numerical solution).

$\frac{dR(\lambda)}{d\lambda} = 0$ Find λ_{max} wavelength at which the peak occurs.

Wien's law: $\lambda_{max} = \frac{2.898x10^{-3} m{\cdot}K}{T}$

b. Integrate Planck's formula to obtain the Stefan-Boltzmann law. Use the definite integral:

$$\int_0^\infty \frac{x^3}{(e^x - 1)}\,dx = \frac{\pi^4}{15}$$

$I = \frac{P}{A} = \int_0^\infty R(\lambda)\,d\lambda$ Find the total $\frac{P}{A}$ over all wavelengths.

Stefan-Boltzmann law: $I = \frac{P}{A} = \sigma T^4$

Do not use a symbolic math program to derive the answers, except in determining Wien's law. You may use a symbolic math program or numeric methods to determine the solution for the non-analytical step. After differentiating and setting equal to zero, let $x = {}^{hc}/_{\lambda kT}$ and the equation reduces to $1 - e^{-x} = \frac{x}{5}$ which must be solved numerically. Perhaps this has something to do with Wien's law's constant not having a special symbol of its own?