From the Earth to Mars - Spaaaaaaaaaaaaace Cadets!

Exercise 2:

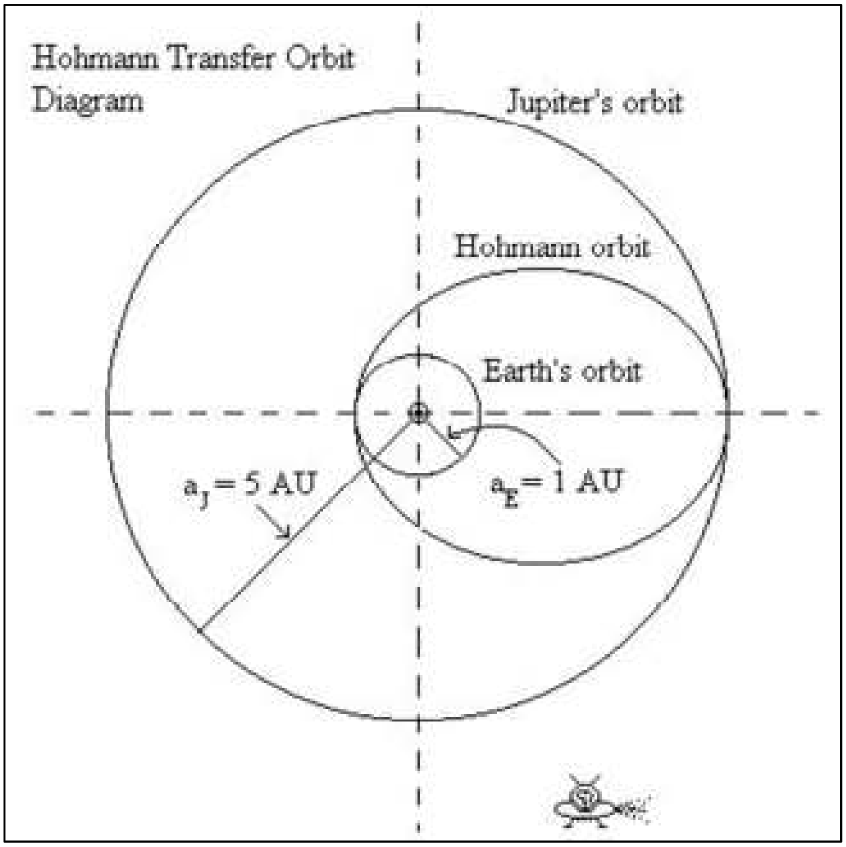
In this exercise we will use Kepler’s laws (and a few other equations) to determine the trajectory, time of flight, and the different velocity changes a space probe would need to perform for a flight from the Earth to Mars using a minimum energy transfer orbit (the Hohmann orbit). See the next page for a list of equations, constants, and definitions you will need to complete the “journey”.

Assume circular orbits for Earth and Mars. Determine the semi-major axis length and eccentricity of the Hohmann transfer orbit between them. Use Kepler’s 3rd law to determine the flight time from Earth’s orbit to Mars’ orbit. Using Earth’s and Mars' circular orbit speeds, the escape velocity from Earth's surface, the vis-viva equation for the Hohmann orbit, and conservation of energy considerations in the vicinity of Mars to determine all the "delta-vee's" (Δv) for a trip from Earth to Mars. Here is a list of the "delta-vee's":

1. Δv (1) = escape speed from Earth's surface: assume that once the spacecraft reaches 100 Earth radii directly ahead of the surface of Earth. It is now in orbit around the Sun with the same speed as the Earth's circular orbital speed.

2. Δv (2) = change in speed from Earth's circular orbital speed to the perihelion speed of the Hohmann orbit: spacecraft begins its long "climb" to Mars' orbit distance.

3. Δv (3) = change in speed necessary to soft-land on Mars: assume your trajectory is such that the spacecraft arrives 100 Mars' radii directly ahead of the surface of Mars at the instant of Hohmann orbit aphelion. Use the following procedure: determine the difference between the Hohmann orbit aphelion speed and Mars' circular orbit speed (this would be the closing speed between the spacecraft and Mars at the point when the spacecraft is 100 Mars' radii above the surface). Now use energy conservation to determine the speed at which the spacecraft would hit the surface, ignoring atmospheric friction. This "impact speed" should be the "delta vee" necessary to slow the spacecraft down for a soft landing on the surface.



Graphic of Earth – Jupiter Hohmann transfer orbit. Hohmann orbit is just tangent to Earth's orbit at its perihelion and just tangent to Jupiter's orbit at its aphelion.

Ellipse geometry: $r_p = a(1-e)$, $r_a = a(1+e)$, $r_p \equiv$ perihelion distance, $r_a \equiv$ aphelion distance

Vis-viva equation, also referred to as orbital energy conservation equation (*Vis viva* = Latin for "live force" is a term from the history of mechanics.):

$v^2 = GM(2/r - 1/a)$; or for perihelion speed & aphelion speed (note: $T \equiv$ orbital period):

perihelion speed: $v_p^2 = GM(1+e)/a(1-e) = 4\pi^2 a^2(1+e)/T^2(1-e)$

aphelion speed: $v_a^2 = GM(1-e)/a(1+e) = 4\pi^2 a^2(1-e)/T^2(1+e)$

Escape velocity: $v_{esc} = (2GM/r)^{-½}$, from radius r (note: $M \equiv$ mass of body being "escaped" from)

$M_{\odot} = 2 \times 10^{30}$ kg, $M_{earth} = 6 \times 10^{24}$ kg, $R_{earth} = 6400$ km, $M_{mars} = 0.1 M_{earth}$, $R_{mars} = 3400$ km

$a_{earth} = 1.5 \times 10^{11}$ m = 1 AU, $a_{mars} = 1.5$ AU