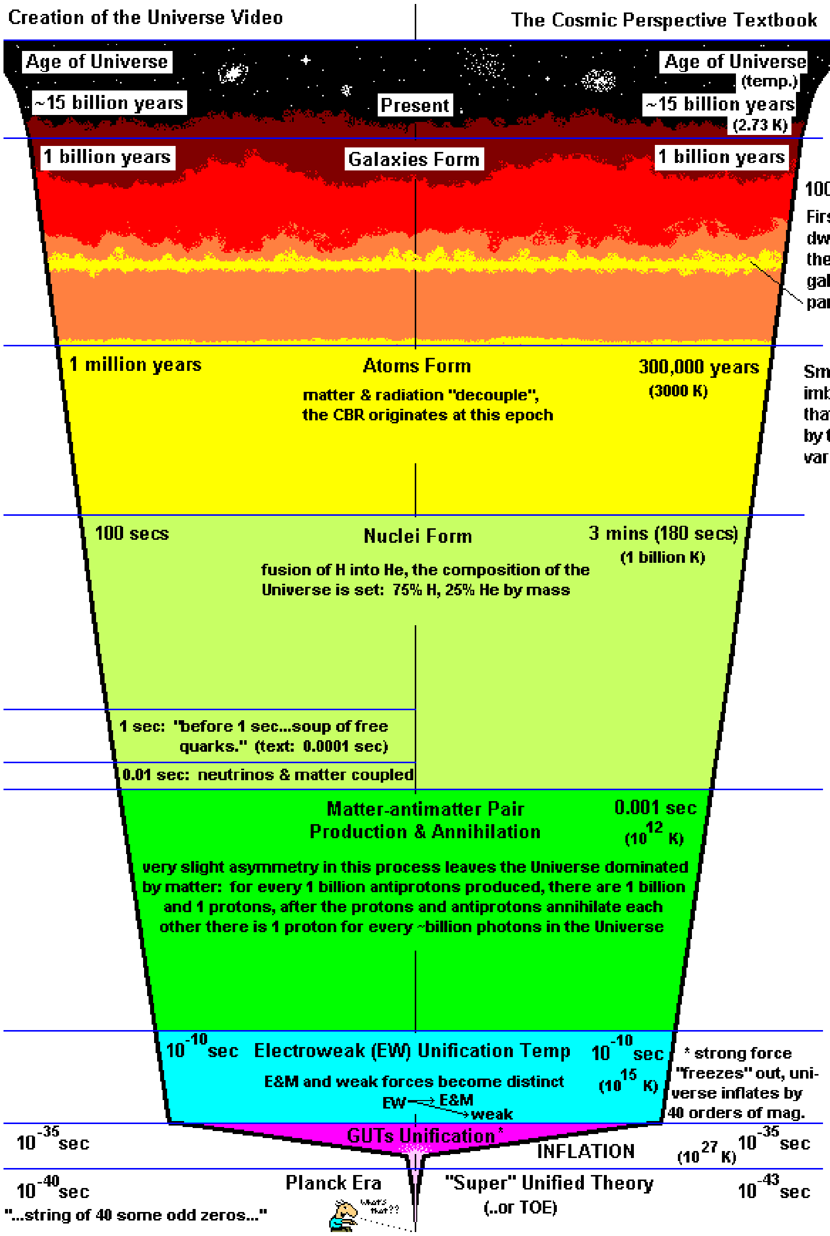
The Last Note of the Music From the Beginning of Time: Determining the curvature of the Universe from variations in the Cosmic Microwave Background (CMB)



Some Recent Numbers:

Universe is accelerating in its expansion!

Age of the Universe: 13.7 +/- 0.2 billion years

100 to 400 million years: First generation of stars form in small dwarf-irregular like galaxies, which then fall together to form larger galaxies. These early stars cause a partial reionization of the universe.

Small angular variations are imbedded into the CBR at the time that matter & radiation "decouple" by the pre-existing slight density variations within the plasma.

GUTs = Grand Unified Theories
Theories that unify the 3 forces described by quantum mechanics.
TOEs = Theories of Everything
Unifying theories that include the gravity force (most likely "quantum gravity").
[Also called: "Super" Unified Theories.]

String Theory vies for best candidate for "Super" Unified Theory.

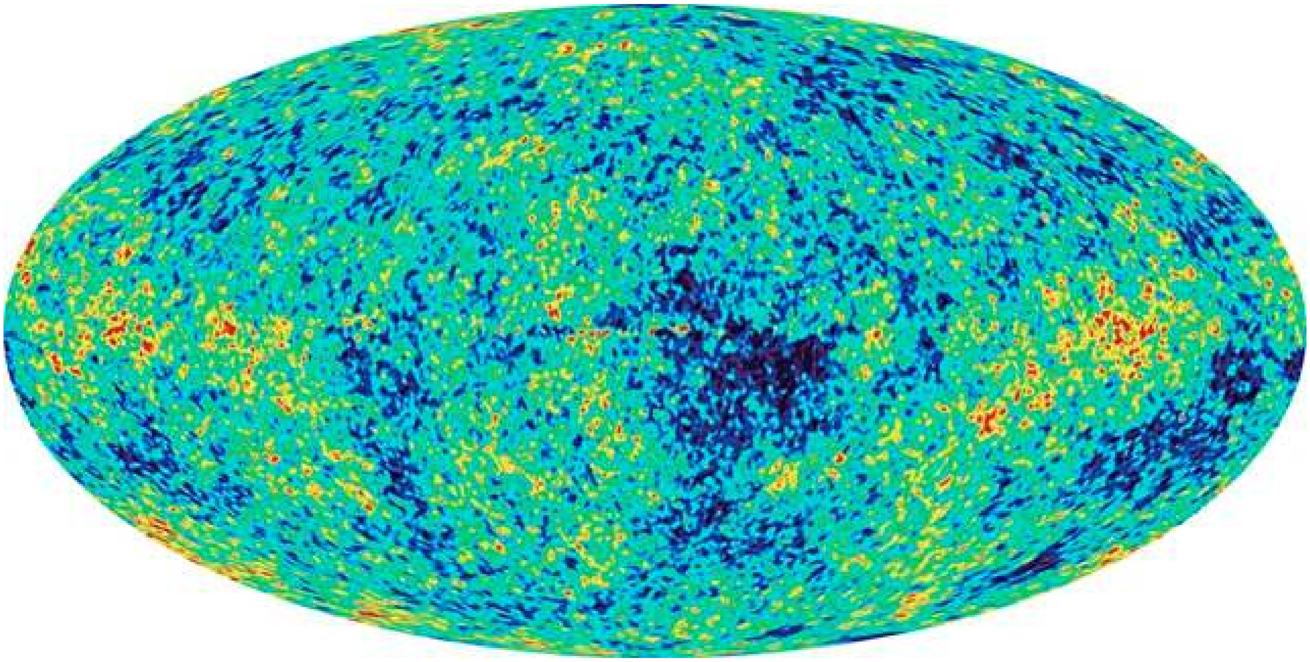
The CMB or cosmic background radiation (CBR) ruled out the Steady State cosmology which did NOT predict the CBR. However, there were 2 problems with the observed CMB;

1. So isotropic that it indicated regions that could not have been in thermal contact were somehow in thermal equilibrium? "horizon problem"

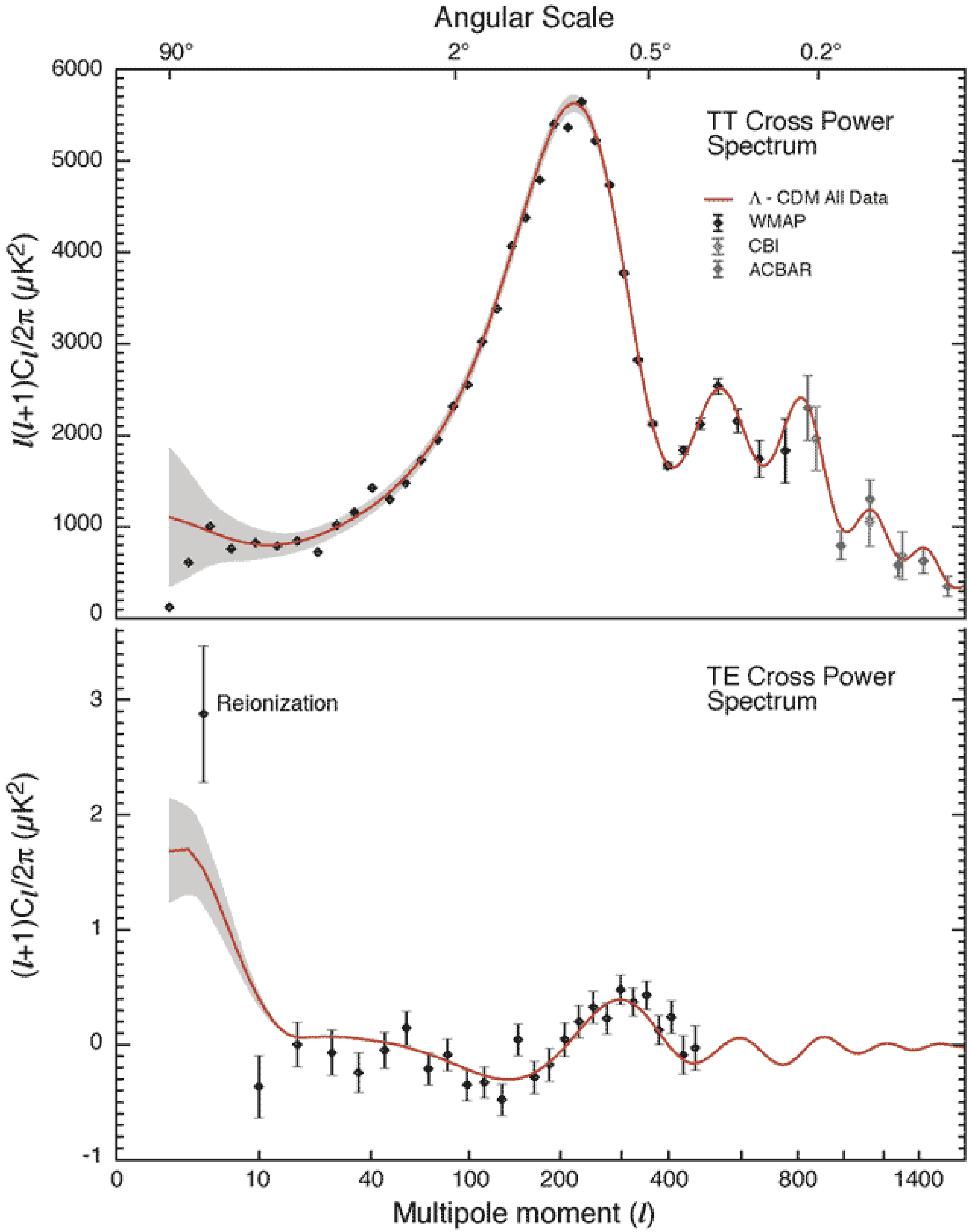
2. So uniform that the "density variations" in the early "plasma" stage of the Universe (that should show up as slight variations in the CMB) that are needed to explain the large scale structure of the present Universe were not observed?

Problem 1 may be explained by a very early extremely rapid expansion of the Universe ("inflation") predicted by some GUTs when the Strong Force "freezes" out of the Grand Unified Force creating a very large energy density for a very short time. (This energy density is related to the Cosmological Constant in Einstein's equations.)

Problem 2 has been resolved by extremely accurate and precise observations of the CMB. See the WMAP all-sky graphic of the CMB variations on the next page:



The above figure shows the actual temperature = brightness = plasma density variations observed by the WMAP probe.
See next page for a figure showing the "spectrum" of angular sizes of the variations. This data can be used to determine many cosmological parameters. The largest peak determines the curvature of the Universe.


Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi\ (\mu K^2)$
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
TE Cross Power Spectrum
Reionization
3
2
1
0
-1
$(l+1)C_l/2\pi\ (\mu K^2)$
0
10
40
100
200
400
800
1400
Multipole moment (l)

Determining the curvature of the Universe from the variations in the CMB.

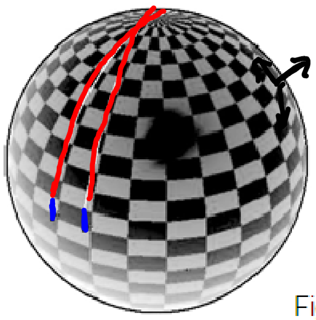


Figure 1. a.

Closed - || beams converge

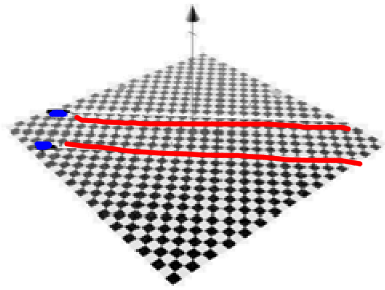


Figure 1. b.

Flat - || beams stay ||

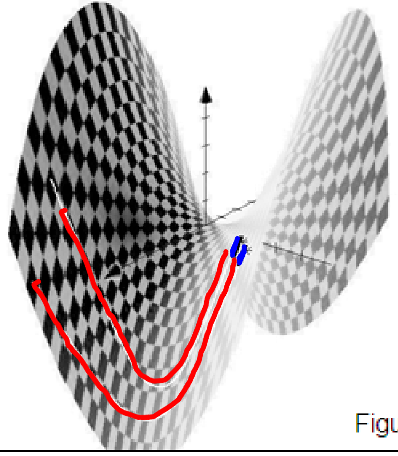
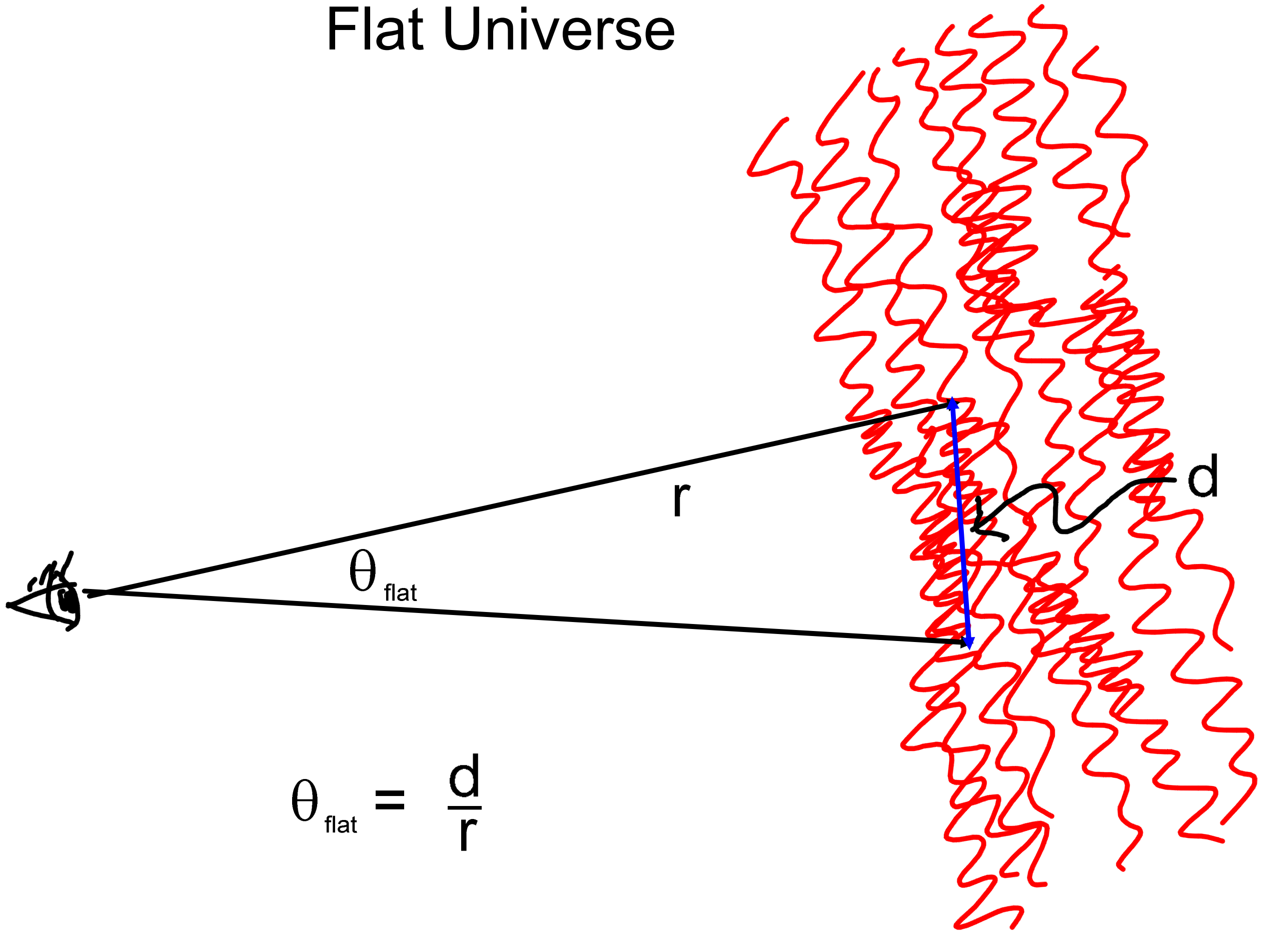


Figure 1. c.

Open - || beams diverge

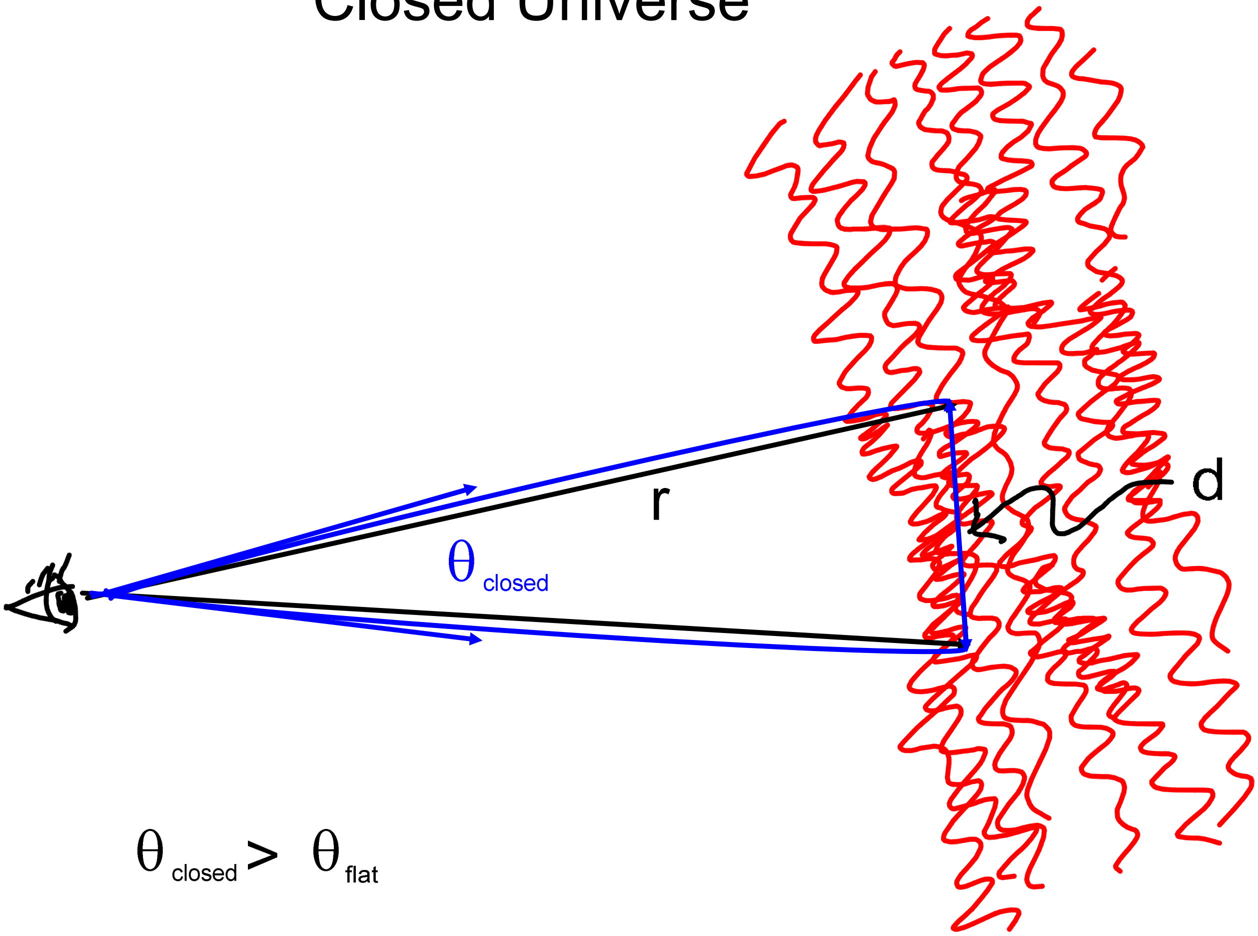
Flat Universe



$$\theta_{flat} = \frac{d}{r}$$

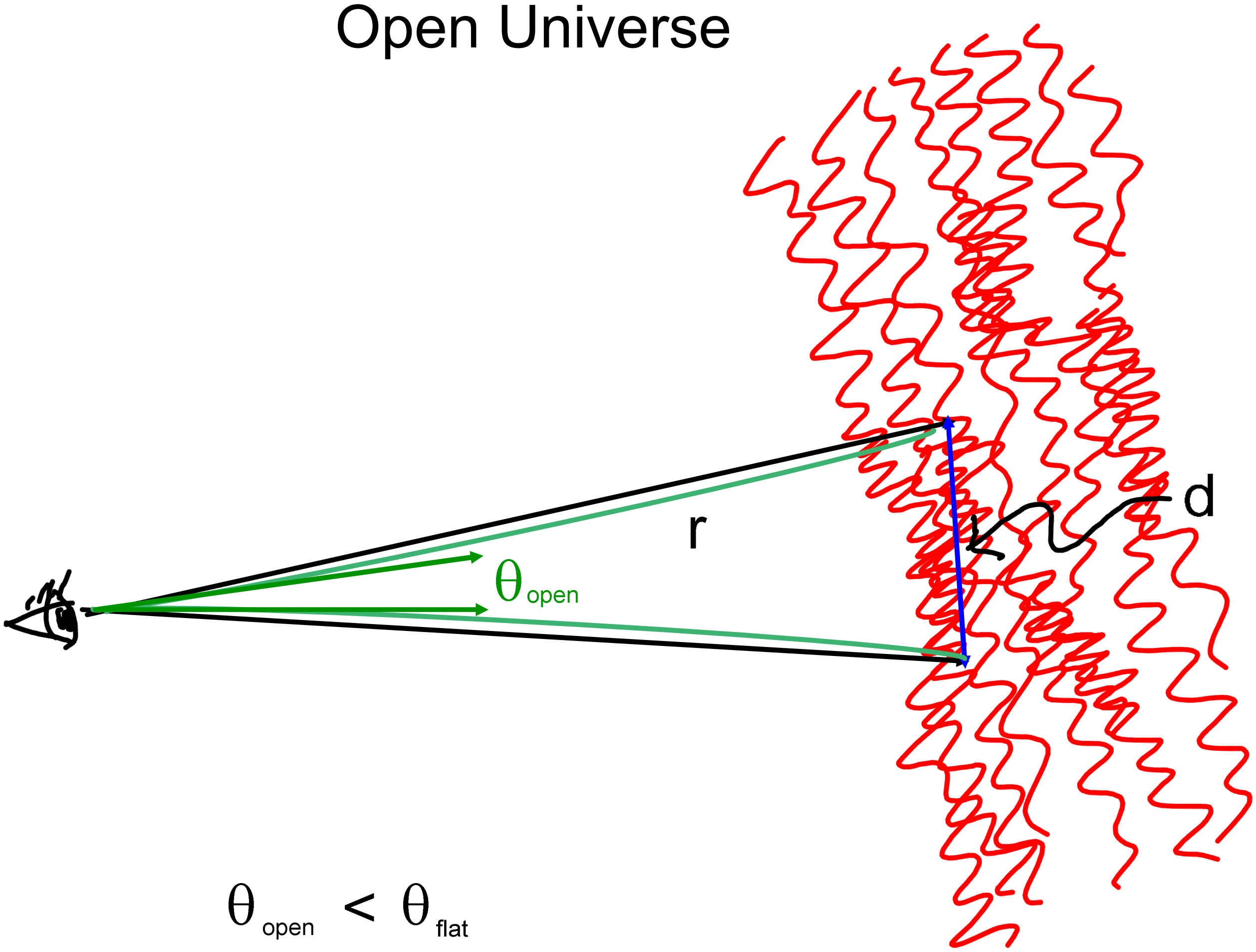
The physics of plasma waves and "boundary conditions" for the time in which this physics operates for a given cosmological model lead to predictions about the physical size of the density variations at the time that matter and radiation "decouple". These variations are "imbedded" in the CMB as the angular variation that WMAP (and other projects) measured.

Closed Universe



$\theta_{\text{closed}} > \theta_{\text{flat}}$

Open Universe



$\theta_{open} < \theta_{flat}$

Size of temperature variations in the plasma (relativistic gas of baryons and photons):

equation of state for plasma: $P = \frac{1}{3}\rho c^2$

speed of sound in the plasma: $c_s = \sqrt{\frac{dP}{d\rho}} = \frac{c}{\sqrt{3}}$

The density variations in the dark matter (set by the particular inflation model) attract the baryon-photon fluid (plasma) which compresses, then rebounds starting a periodic contraction and expansion at the various regions in the fluid (the "sound waves" begin everywhere in phase).

Consider a particular oscillation starting as an over-dense or under-dense region. By the time of recombination, let this oscillation complete 1/2 period so now it would be an under-dense or over-dense region (opposite of its start). The corresponding wavelength of this oscillation:

$\tau = \frac{\lambda}{c_s}$ $\frac{1}{2}\tau = t_{rec}$

$$\lambda = 2c_s t_{rec} = \frac{2ct_{rec}}{\sqrt{3}}$$

These regions will be the "spots" on the CMB that are higher or lower temperature than the mean.

This leads fo the fluctuations in the power spectrum of the CMB sky that will have discrete peaks at these favored spatial scales.

The analysis of the peaks can determine most of the parameters describing a cosmological model.

The angular scale of the first acoustic peak is a measure of the global curvature of space. The physical scale of the first peak is called the "sound-crossing horizon" at the time of recombination. This physical scale makes an excellent "standard ruler".

$D_s = \lambda \left(\frac{1}{2}\tau = t_{rec}\right)$ sound crossing horizon

$$D_s = \frac{2ct_{rec}}{\sqrt{3}} = \frac{2 \times 1\frac{ly}{y} \times 400{,}000y}{\sqrt{3}} = 140\,kpc$$

or in terms of the redshift

$D_s = \frac{2ct_0}{\sqrt{3}}(1+z)^{-\frac{3}{2}}$ $\quad t_0 \equiv$ present age of the Universe

The angle subtended by the region = $\dfrac{\text{size of the region}}{\text{distance to us (at the time of emission)}}$

$$\theta = \frac{D_S}{D_A}$$

Must use the equations resulting from solving the FRW metric to get the correct "angular-diameter distance", then for k = 0 (flat):

$$\theta = \frac{D_S}{D_A} = \frac{2ct_0(1+z_{rec})^{-3/2} \leftarrow 140\,\text{kpc}}{3ct_0\left[(1+z_{rec})^{-1} - (1+z_{rec})^{-3/2}\right] \leftarrow 12{,}000\,\text{kpc}}$$

prediction for flat Universe:

$z_{rec} \approx 1100$

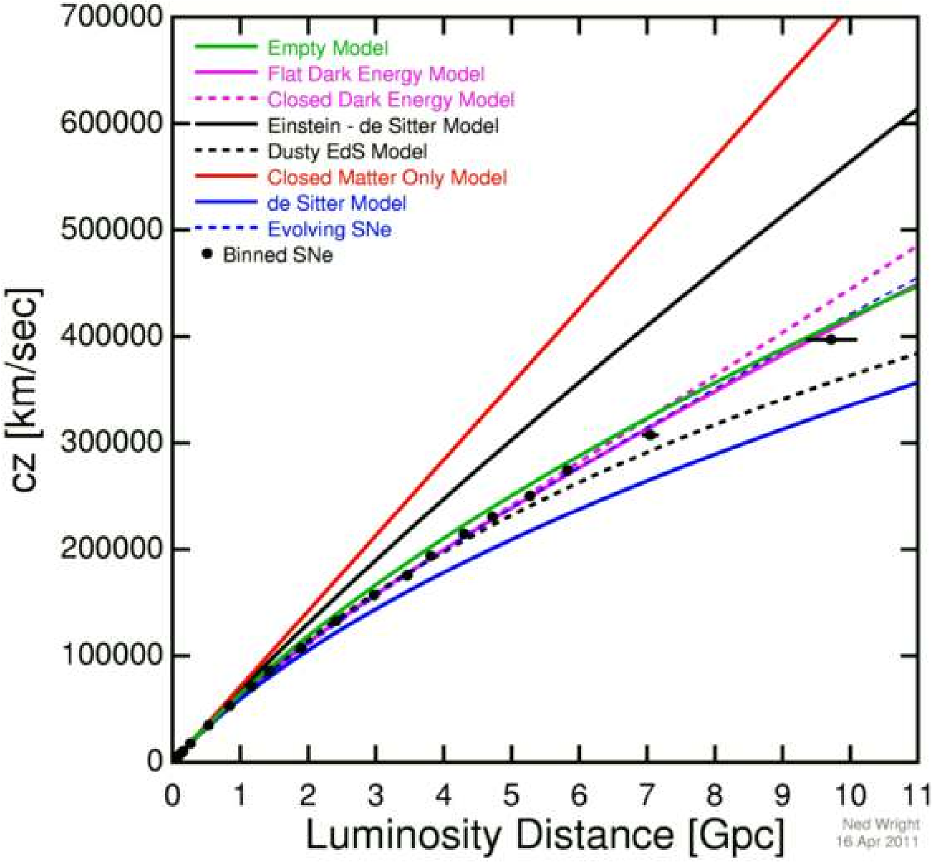
$\theta \approx 0.012\ \text{rad} = 0.7^\circ$

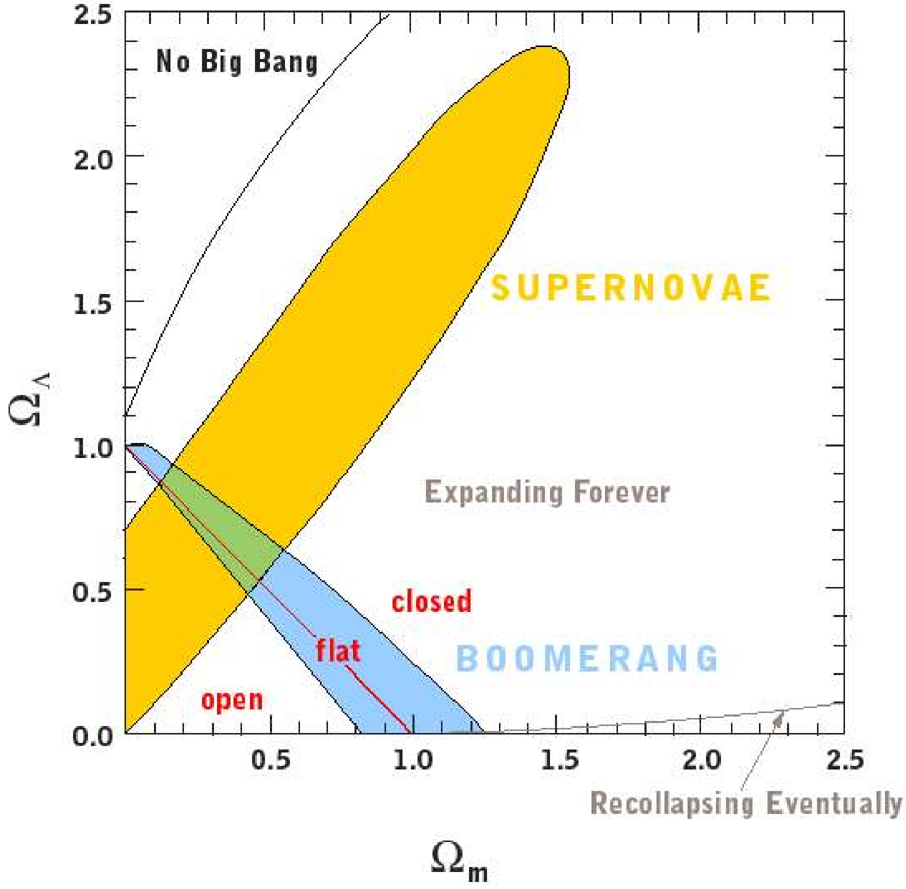
Results of the analysis of the angular size spectrum of the CMB:

$\Omega_m + \Omega_\Lambda = 1.02 \pm 0.02$ flat

$\Omega_m \approx 0.3 \quad \Omega_\Lambda \approx 0.7$

$t_0 \approx 13.7 \pm 0.2\,\text{Gyr} \quad \Omega_B = 0.044 \pm 0.004$


700000
600000
500000
400000
300000
200000
100000
0
cz [km/sec]
0
1
2
3
4
5
6
7
8
9
10
11
Luminosity Distance [Gpc]
Empty Model
Flat Dark Energy Model
Closed Dark Energy Model
Einstein - de Sitter Model
Dusty EdS Model
Closed Matter Only Model
de Sitter Model
Evolving SNe
● Binned SNe
Ned Wright
16 Apr 2011


No Big Bang
SUPERNOVAE
Expanding Forever
closed
flat
open
BOOMERANG
Recollapsing Eventually
Ω_Λ
Ω_m
0.0
0.5
1.0
1.5
2.0
2.5