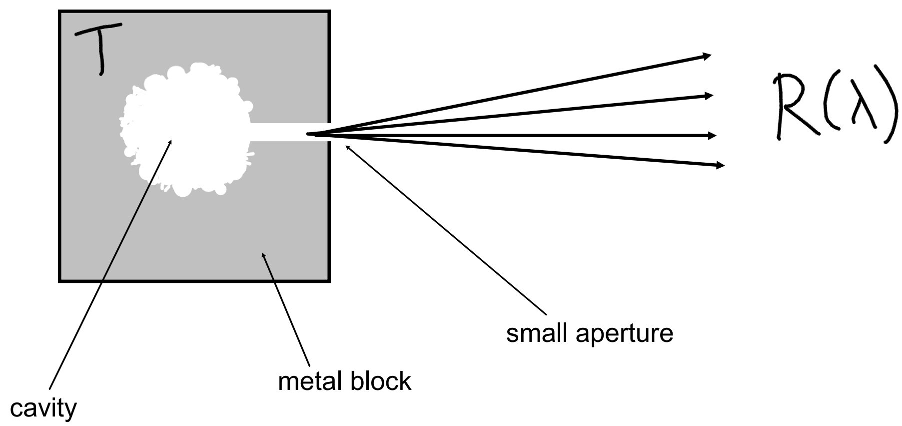
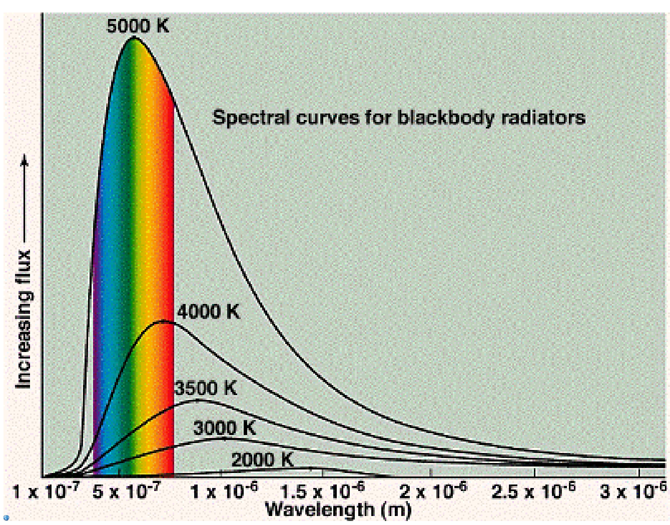
Blackbody Radiation

Cavity Radiation = closest "lab source" to perfect thermal radiation (blackbody radiation)



Characteristics of the blackbody or cavity radiation spectrum:



(empirically derived laws)

Stefan's law (Stefan-Boltzmann law): [~1880]

$$I = \frac{P}{A} = \sigma T^4$$

Wien's law: [1893]

$$\lambda_{max} = \frac{2.898 \times 10^{-3}\, m \cdot K}{T}$$

T is in absolute temp (eg: kelvin)

Classical analysis of cavity radiation (Rayleigh-Jeans formula):

Assume the cavity is filled with standing E&M waves from "atomic oscillators" in the walls of the cavity. The oscillators have a continuous distribution of energies determined by the temperature of the metal.

$R(\lambda)$ = radiancy (essentially intensity as a function of wavelength)

$$R(\lambda) = \frac{c}{4} u(\lambda)$$

where:

$u(\lambda)$ = energy density in the cavity volume as a function of wavelength

$$u(\lambda) = \frac{N(\lambda)}{V} \langle E \rangle$$

where:

$\langle E \rangle$ = total energy per standing wave = total energy per atomic oscillator

V = volume of cavity

$N(\lambda)$ = number of standing waves in the cavity as a function of wavelength

$$N(\lambda) = \frac{8\pi V}{\lambda^4}$$

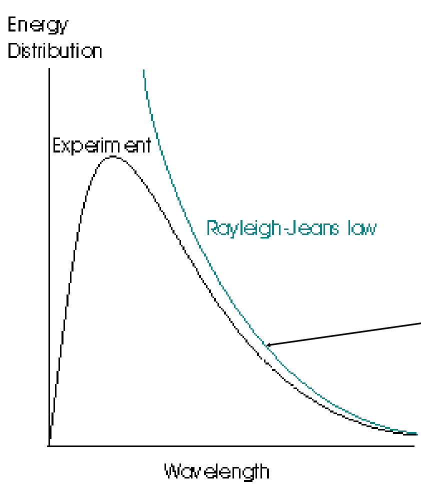
derived for E&M waves in a cubical cavity, deviation not shown from classical thermodynamics: $\langle KE \rangle_{osc} = \frac{1}{2}kT$

from the Virial Theorem applied to the harmonic oscillator potential: $\langle KE \rangle_{osc} = \langle PE \rangle_{osc}$

$$\langle E \rangle = \langle KE \rangle_{osc} + \langle PE \rangle_{osc} = kT$$

So the classical derivation (Rayleigh-Jeans formula):

$$R(\lambda) = \frac{c}{4}u(\lambda) = \frac{c}{4}\frac{N(\lambda)}{V}\langle E \rangle = \frac{c}{4}\frac{8\pi V}{\lambda^4}\frac{1}{V}kT$$



the ultraviolet catastrophe

$$\boxed{R(\lambda) = \frac{2\pi c k T}{\lambda^4}}$$

Rayleigh-Jeans formula

Planck's solution to the UV catastrophe:

He postulated that the "atomic oscillators" could only absorb or emit energy in discrete bundles (or quanta) where the energy per quanta is proportional to the frequency. This effectively limited the high frequency (short wavelength) contributions to the energy emitted and gave the correct formula for cavity (blackbody) radiation.

E = energy absorbed or emitted per oscillator

$$E = n\varepsilon \quad n = 1, 2, 3, \ldots$$

and $\varepsilon = hf$ (energy per quanta)

$h \equiv$ Planck's constant

$$h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}$$

"Quantizing" the energy of the atomic oscillators in the classical analysis results in a formula that correctly describes the blackbody (cavity radiation) spectrum:

Planck's Equation:

$$\boxed{R(\lambda) = \left(\frac{c}{4}\right)\left(\frac{8\pi}{\lambda^4}\right)\left[\left(\frac{hc}{\lambda}\right)\frac{1}{e^{hc/\lambda kT} - 1}\right]}$$