Eclipsing-spectroscopic binary systems

spectroscopic binary = one or both components' spectra can be used to determine radial velocity curves for the components

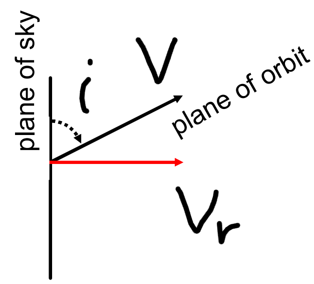
eclipsing binary = components periodically eclipse one another

eclipsing-spectroscopic binary = both light curve and velocity curve data measurable

spectroscopic binary systems

Observational definitions: velocity curve = plot of radial velocity of one or both components as a function of time, inclination = angle of true orbital plane to plane of sky (i = 0 deg: planes conincide; i = 90 deg: orbital plane edge on to line of sight

radial velocity: line of sight component of actual orbital velocity:



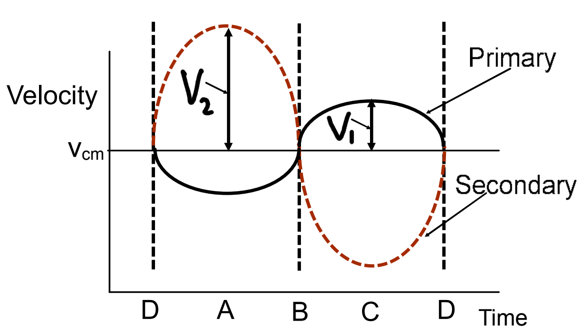
$$V_r = V \sin i$$

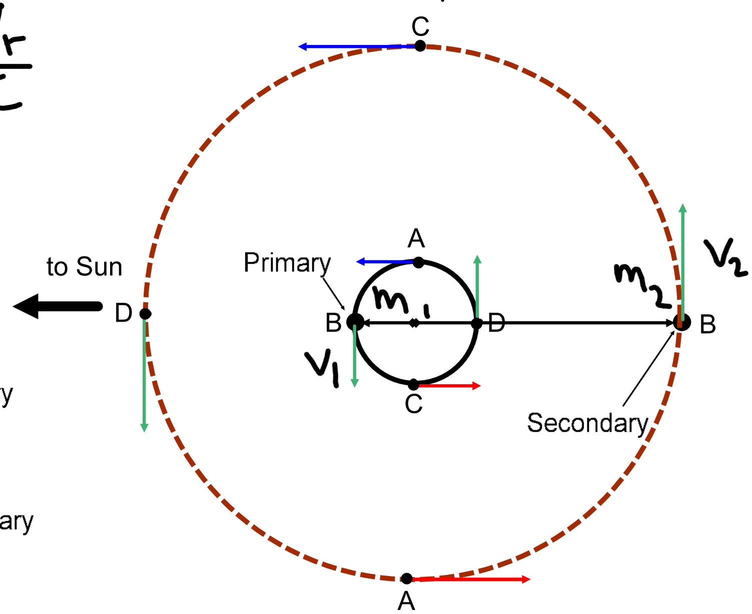
Determine radial velocity by measuring the Doppler shift:

$$\frac{\Delta\lambda}{\lambda_0} \equiv \frac{(\lambda - \lambda_0)}{\lambda_0} = \frac{V_r}{c}$$

Example - circular orbits, i = 90°

Velocity Curve



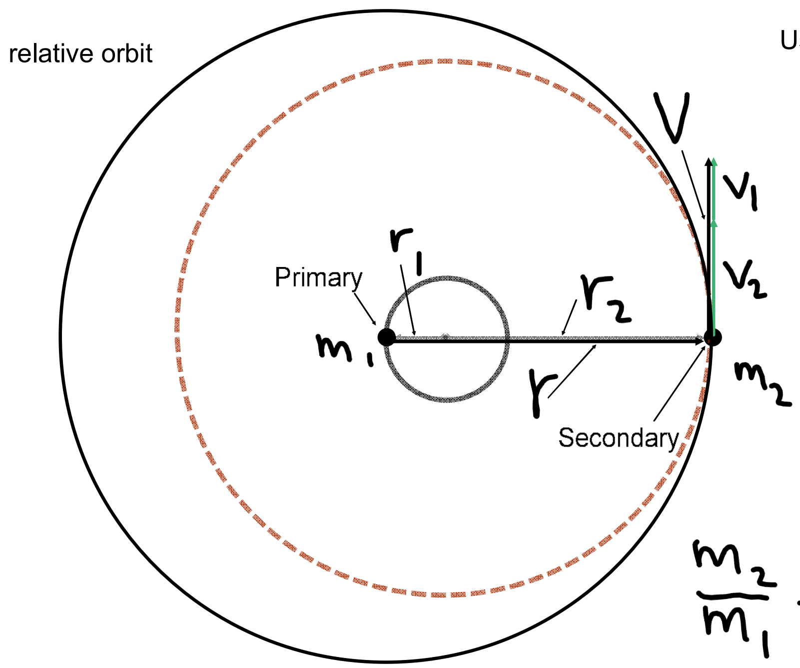


Spectroscopic binary determination of the mass:

NOTE: m_1 = primary, m_2 = secondary

Example - circular orbits, $i = 90^{o}$

$$V = V_1 + V_2 \qquad r = r_1 + r_2$$



Use V to determine r from:

$$V = \frac{2\pi r}{P}$$

From center of mass:

$$m_1 r_1 = m_2 r_2$$

From conservation of momentum:

$$m_1 v_1 + (-m_2 v_2) = 0$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{m_2}{m_1} = \frac{V_1}{V_2} = \frac{r_1}{r_2} = \frac{V_{1r}}{V_{2r}}$$

Ratio of the masses from:

$$\frac{m_2}{m_1} = \frac{V_1}{V_2} = \frac{V_{1r}}{V_{2r}}$$

Sum of the masses from K3:

$$m_1 + m_2 = \frac{4\pi^2 r^3}{G P^2}$$

V_1, V_2, P all determined from velocity curve *if inclination is known*

Now from the known sum of the masses and the known ratio of the masses, solve 2 equations, 2 unknowns to determine the individual masses of the components: m_1 & m_2

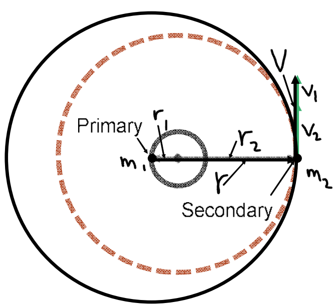
If the inclination is not known, we cannot get the individual masses, but only the ratio of the masses.

We also assumed a circular orbit (which most short period spectroscopic binaries would have due to tidal interactions between the components). Non-circular orbits present distinct velocity curve shapes that can be analyzed to determine the eccentricity & orientation of the major axis to the line-of-sight. Check out this link: http://www.astro.cornell.edu/academics/courses/astro1101/java/binary/binary.htm for a computer simulation demo of a double lined spectroscopic binary. You can change all the parameters (component masses, semimajor axis of the relative orbit, eccentricity, inclination, etc.) to see the effect on the velocity curve.

Extrasolar Planet Example (Single line spectroscopic binary with a low mass secondary in a circular orbit):

Radial Velocity Detection Method

relative orbit



For this case:

a) $m_1 \gg m_2 \rightarrow m_1 + m_2 \cong m_1$

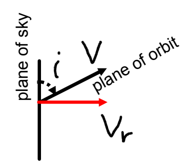
b) $r_2 \gg r_1 \rightarrow r = r_1 + r_2 \cong r_2$

c) $V_2 \gg V_1 \rightarrow V_1 + V_2 \cong V_2$

Measured or inferred quantities:

$$\boxed{V_{1r} = V_1 \sin i \; ; \; P \; ; \; m_1}$$

note:



① $V_1 + V_2 = \frac{2\pi r}{P} \rightarrow \boxed{V_2 = \frac{2\pi r_2}{P}}$ using b) & c) above

② $\boxed{\frac{m_2}{m_1} = \frac{V_1}{V_2} = \frac{V_{1r}}{V_{2r}}}$

③ $m_1 + m_2 = \left(\frac{4\pi^2}{G}\right)\frac{r^3}{P^2} \xrightarrow{\text{using a) \& b) above:}} \boxed{m_1 = \left(\frac{4\pi^2}{G}\right)\frac{r_2^3}{P^2}}$

Now solve for m_2 mass of planet

Solve modified equation 3 for $r \cong r_2$

$$r_2^3 = \left(\frac{G}{4\pi^2}\right) m_1 P^2 \qquad r_2 = \left(\frac{G}{4\pi^2}\right)^{\frac{1}{3}} m_1^{\frac{1}{3}} P^{\frac{2}{3}}$$

Use r_2 to solve for V_2 in modified equation 1

$$V_2 = \frac{2\pi}{P} r_2 = \frac{2\pi}{P}\left(\frac{G}{4\pi^2}\right)^{\frac{1}{3}} m_1^{\frac{1}{3}} P^{\frac{2}{3}}$$

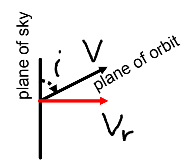
$$V_2 = (2\pi G)^{\frac{1}{3}} m_1^{\frac{1}{3}} P^{-\frac{1}{3}}$$

Use equation 2 to solve for m_2 (mass of the planet)

$$m_2 = m_1 \frac{V_1}{V_2} = m_1 V_1 (2\pi G)^{-\frac{1}{3}} m_1^{-\frac{1}{3}} P^{\frac{1}{3}}$$

$$m_2 = \left(\frac{m_1^2 P}{2\pi G}\right)^{\frac{1}{3}} V_1$$

Now in general, the inclination is not known, so:



$$V_1 = \frac{V_{1r}}{\sin i}$$

$$\boxed{m_2 \sin i = \left(\frac{m_1^2 P}{2\pi G}\right)^{\frac{1}{3}} V_{1r}}$$

All we can find is a lower bound to the mass of the planet, the so-called "mass-sin(i)"

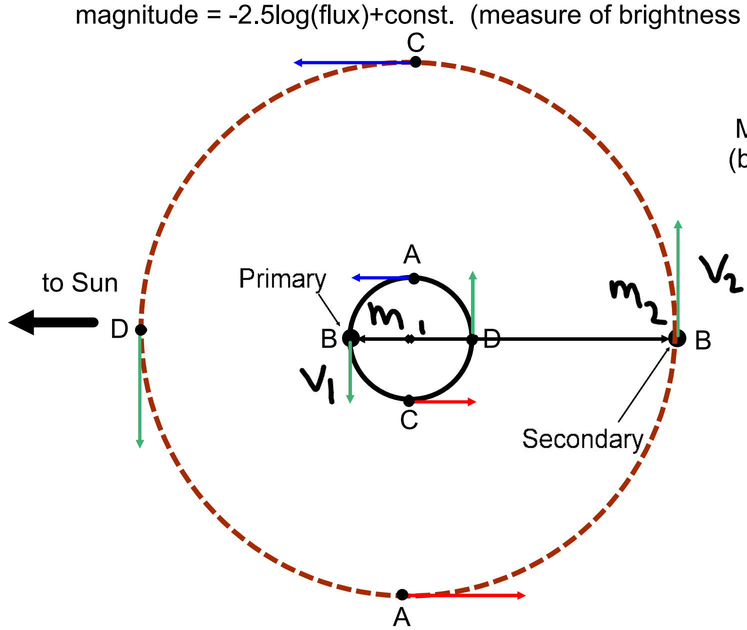
If mass is given in solar masses ($M_\odot$), velocities in km/s, and period in years, then the equatin may be written:

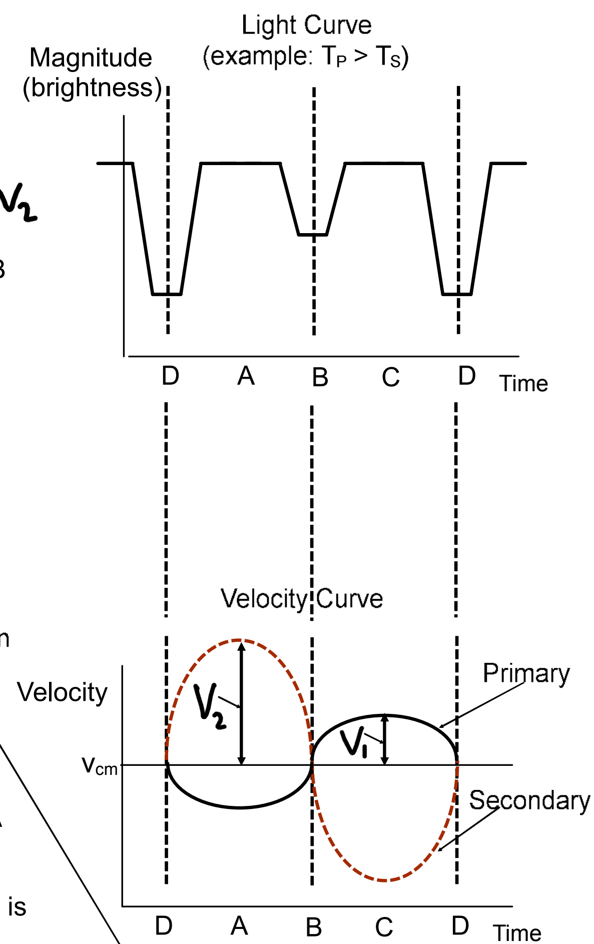
$$m_2 \sin i = \left(\frac{m_1^2 P}{30}\right)^{\frac{1}{3}} V_{1r}$$

eclipsing binary systems

Observational definitions:

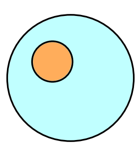
light curve = plot of combined brightness of the unresolved components as a function of time
inclination = angle of true orbital plane to plane of sky (by definition, the eclipsing binary system's orbital plane must be very close to edge on to line of sight in order for mutual eclipses to occur: i = ~90 degrees)
magnitude = -2.5log(flux)+const. (measure of brightness for astronomers)





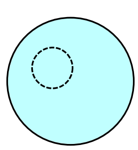
For a larger radius primary with a surface temp greater than a smaller radius secondary:

$R_P > R_S$ and $T_P > T_S$ $P_P/A_P > P_S/A_S$ (from S-B law)

D: 

area of primary occulted = A_P = A

net power occulted is greater for the hotter component, so eclipse is deeper (position D in example)

B: 

area of secondary occulted = A_S = A
(same area occulted: $A_P = A_S$)

Stefan-Boltzmann law:

$P/A = \sigma T^4$

eclipsing binary systems

Several parameters may be determined from the light curve:

Period (P) of the orbit (usually with more precision than with the velocity curve)

Inclination (i) of the orbit (at least approx. 90 deg, but with measured or inferred properties of the stars & binary orbit, an exact value may be determined)

relative surface temps (T_P/T_S) from relative depths of light curve

stellar radii (at least relative to the orbit semimajor axis, but with spectroscopic binary data the exact value may be determined)

other.... orbital eccentricities, orientation of orbit major axis to line of sight, etc.

Eclipsing-spectroscopic binary systems

Combining information from the spectroscopic data and the light curve the individual masses, sizes (radii), relative surface temperatures, orbital parameters, etc. may be determined

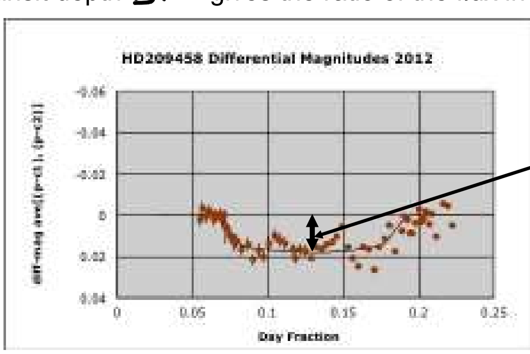
Since there are many more eclipsing-spectroscopic binary systems know (distance to system not necessary to determine stellar parameters) than visual binary systems, these systems are very important for determining the true nature and characteristics of stars

Extrasolar Planet Example (continued) (Single line spectroscopic binary with a low mass secondary in a circular orbit - with transits of the planet observed via light curve):

Transit Detection Method

Light curve data combined with the spectroscopic information allows a more complete determination of the characteristics of the planet. Combined with measured or inferred information about the parent star (primary for the system), the exact inclination and semimajor axis length of the planets orbit, the mass of the planet, and the radius of the planet may be determined.

The transit depth Δm gives the ratio of the flux in transit $\frac{I_{*T}}{I_*}$ to the flux out of transit of the parent star

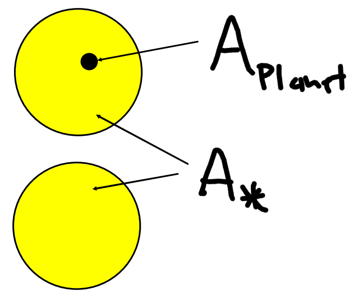


Δm

$$\frac{I_{*T}}{I_*} = 10^{-\frac{\Delta m}{2.5}}$$

$$I_{*T} \propto \left(\left(\frac{P}{A}\right)_* A_* - \left(\frac{P}{A}\right)_* A_{Planet}\right)$$

$$I_* \propto \left(\frac{P}{A}\right)_* A_*$$



$$\frac{I_{*T}}{I_*} = \frac{\left(\left(\frac{P}{A}\right)_* A_* - \left(\frac{P}{A}\right)_* A_{Planet}\right)}{\left(\frac{P}{A}\right)_* A_*}$$

$$\frac{I_{*T}}{I_*} = 1 - \frac{A_{Planet}}{A_*} = 1 - \left(\frac{R_{Planet}}{R_*}\right)^2 \qquad A = \pi R^2$$

$$\frac{R_{Planet}}{R_*} = \sqrt{1 - \frac{I_{*T}}{I_*}} = \sqrt{1 - 10^{-\frac{\Delta m}{2.5}}}$$