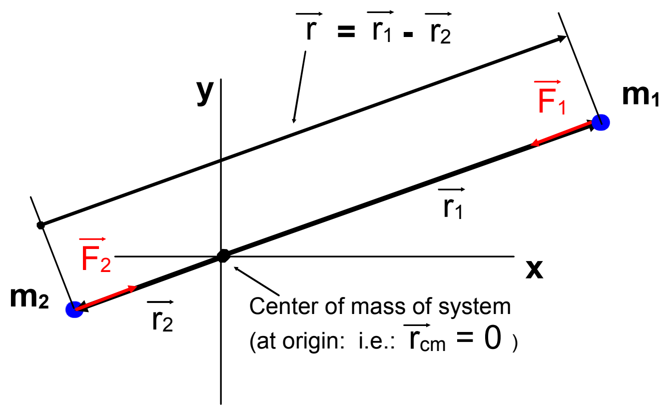
Setup for solving the "two body problem"

$$\vec{F}_1 = -\frac{G m_1 m_2}{r^2}\hat{e}_r \quad \left(\hat{e}_r = \frac{\vec{r}}{r}\right)$$



Reduce 2-body problem to "motion under central force", in this case, "central force" will be Newton's inverse square law for gravity.

$$m_1\vec{r}_1 + m_2\vec{r}_2 = (m_1+m_2)\vec{r}_{cm}$$ From definition of center-of-mass

$$\vec{r}_{cm} = 0$$ From setup

$$m_1\vec{r}_1 + m_2\vec{r}_2 = 0$$

$$\vec{r}_2 = -\frac{m_1}{m_2}\vec{r}_1 \qquad \text{also } \vec{r} = \vec{r}_1 - \vec{r}_2$$ Eliminate $\vec{r}_2$

$$\vec{r} = \vec{r}_1 + \frac{m_1}{m_2}\vec{r}_1 = \vec{r}_1\left(1 + \frac{m_1}{m_2}\right) = \vec{r}_1\left(\frac{m_1+m_2}{m_2}\right)$$

Setup the D.E. (from Newton's 2nd law) of motion for m_1 relative to c.m.:

$$\boxed{m_1 \ddot{\vec{r}}_1 = -\frac{G m_1 m_2}{r^2}\hat{e}_r}$$

note: $\vec{r} = \vec{r}_1 \left(\frac{m_1+m_2}{m_2}\right)$

$$\dot{\vec{r}} = \dot{\vec{r}}_1\left(\frac{m_1+m_2}{m_2}\right)$$

$$\ddot{\vec{r}} = \ddot{\vec{r}}_1\left(\frac{m_1+m_2}{m_2}\right) \Rightarrow \ddot{\vec{r}}_1 = \frac{m_2}{m_1+m_2}\ddot{\vec{r}}$$

Replace $\ddot{\vec{r}}_1$ with $\frac{m_2}{m_1+m_2}\ddot{\vec{r}}$ (In the D.E. above.)

$$\frac{m_1 m_2}{m_1+m_2}\ddot{\vec{r}} = -\frac{G m_1 m_2}{r^2}\hat{e}_r$$

let $\mu = \frac{m_1 m_2}{m_1+m_2} \equiv$ "reduced mass"

$$\mu \ddot{\vec{r}} = -\frac{G m_1 m_2}{r^2}\hat{e}_r$$ D.E. in terms of $\vec{r}$

Or finally write D.E. in terms of m_1 and its acceleration relative to m_2:

$$\boxed{m_1 \ddot{\vec{r}} = -\frac{k}{r^2}\hat{e}_r \quad \text{where } k = G(m_1+m_2)m_1}$$