* There is no such thing as a perfect measurement.
- when we take a measurement there is always a limitation on how close we are to the true value
  -- if you measure a mass to the mg than then error in your measurement is at least a ±mg
  -- this implies that the true value is never known and so that the exact error is also “unknowable”
  -- therefore, we are forced to estimate error
- types of errors
  -- illegitimate
    --- deviations from procedure
    --- inability to use a calculator
  -- bias or systematic errors
    --- an error that persists and cannot be caused by chance
    --- your scale always says you weigh 5 lbs heavier
    --- this must be determined by comparison to theory or alternate measurements
  -- random/precision errors
    --- deviations which result from the precision of the method which is utilized
    --- examples: calibration drift, environmental effect, glassware tolerance
    --- sometimes we can reduce these error by generating more data

* Propagation of Error
- used to determine the uncertainty in a quantity which depends on 1 or more independent variables
- mathematically
  \[ F = F(x_1, x_2, \ldots, x_N) \]
  \[
  \frac{\partial F}{\partial x_1} = \left[ \left( \frac{\partial F}{\partial x_1} \right)^2 + \left( \frac{\partial F}{\partial x_2} \right)^2 + \cdots + \left( \frac{\partial F}{\partial x_N} \right)^2 \right]^{1/2}
  \]
  where \( \hat{x}_i \) is the uncertainty in independent variable \( x_i \)
- Examples
  -- Determine the error in the density of an ideal gas
    \[ \rho = \frac{P}{RT} \]
    where \( T \pm \partial T \) and \( P \pm \partial P \)
    \[
    \frac{\partial \rho}{\partial T} = -\frac{P}{RT^2} \quad \frac{\partial \rho}{\partial P} = \frac{1}{RT}
    \]
    \[
    \partial \rho = \left[ \left( \frac{\partial \rho}{\partial T} \partial T \right)^2 + \left( \frac{\partial \rho}{\partial P} \partial P \right)^2 \right]^{1/2} = \left[ \left( \frac{P}{RT^2} \partial T \right)^2 + \left( \frac{1}{RT} \partial P \right)^2 \right]^{1/2}
    \]
  -- Find the momentum for a body with a mass of \( m = 0.53 \pm 0.01 \text{ kg} \) moving at a
velocity \( v = 9.1 \pm 0.3 \text{ m/s} \).

\[ p = mv \]

\[ \hat{p} = \left( \frac{\partial p}{\partial m} \partial m \right)^2 + \left( \frac{\partial p}{\partial v} \partial v \right)^2 = \sqrt{\left(v \partial m \right)^2 + \left(m \partial v \right)^2} = \sqrt{\left(9.1 \times 0.01\right)^2 + \left(0.53 \times 0.3\right)^2} \]

\[ \hat{p} = 0.18 \text{ kg m/s} \]

\[ p = 0.53 \times 9.1 \text{ kg m/s} = 4.8 \pm 0.2 \text{ kg m/s} \]

-- Now, let’s figure out the error in our equilibrium bond length

\[ B_{[0]} = \frac{h}{8\pi^2 2m_{\text{oxy}} r^2 c} \rightarrow r = \left( \frac{h}{8\pi^2 2m_{\text{oxy}} B_{[0]} c} \right)^{\frac{1}{2}} \times 10^{12} \]

--- before we take this on we need to convert our units:

\[ B_{[0] \text{- m}} = \frac{B_{[0] \text{- cm}} \times 100 \text{ cm}}{\text{m}} \]

\[ \hat{B}_{[0] \text{- m}} = \sqrt{\left( \frac{\partial B_{[0] \text{- mm}}}{\partial B_{[0] \text{- cm}}} \partial B_{[0] \text{- cm}} \right)^2} = 100 \times \hat{B}_{[0]} = 100 \times 0.0001 = 0.1 \text{ m} \]

\[ B_{[0] \text{- m}} = 39.0 \pm 0.1 \text{ m} \]

--- now we are ready as soon as we get everything together

\[ h = 6.6260755 \times 10^{-34} \text{ J s} \]

\[ m = 15.9994 \text{ amu} \times 1.6605402 \times 10^{-27} \text{ kg amu} = 2.65676 \times 10^{-27} \text{ kg} \]

\[ c = 2.99792458 \text{ m/s} \]

\[ \hat{r} = \sqrt{\left( \frac{\partial r}{\partial B_{[0] \text{- m}}} \partial B_{[0] \text{- m}} \right)^2} \]

\[ \hat{r} = \sqrt{\left[ \frac{1}{2} \left( \frac{h}{8\pi^2 c^2 2m_{\text{oxy}} B_{[0] \text{- m}}} \right)^{\frac{1}{2}} \times \left( \frac{-h}{8\pi^2 c^2 2m_{\text{oxy}} B_{[0] \text{- m}}}^2 \partial B_{[0] \text{- m}} \times 10^{12} \right) \right]^2} \]

\[ \hat{r} = 0.15 \text{ pm} \]

\[ r = 116.2 \pm 0.2 \text{ pm} \]

So what do you need to do? You will be computing errors for the situations given below.

1. Use the reduced mass, \( \mu \), and the error of the fundamental frequency, \( \tilde{v} \), to determine the error in the force constant \( k \).
\[ k = \mu \left( 2\pi \nu v_{cm} \right)^2 \quad \text{where} \quad \mu_{AB} = \frac{m_am_B}{m_A + m_B} \]

\[ \tilde{v} = 2900.0 \pm 0.1 \text{ cm}^{-1} \quad 1 \text{ amu} = 1.660540 \times 10^{-27} \text{ kg} \quad c = 2.99792458 \times 10^{10} \text{ cm s}^{-1} \]

\[ m_H = 1.007825 \text{ amu} \quad m_{\text{Cl}} = 34.968853 \text{ amu} \]

2. Determine the error in the rotational partition function using the information given below.

\[ q_{\text{rot}}(T) = \frac{T}{\Theta_{\text{rot}}} \quad T = 298.2 \pm 0.1 \text{ K} \quad \Theta_{\text{rot}} = 15.828 \pm 0.001 \text{ K} \]

3. Determine the error in the vibrational partition function using the temperature given above and the following:

\[ q_{\text{vib}}(T) = \frac{e^{-\Theta_{\text{vib}}/T}}{1-e^{-\Theta_{\text{vib}}/T}} = e^{-\Theta_{\text{vib}}/T} \left(1-e^{-\Theta_{\text{vib}}/T}\right)^{-1} \quad \Theta_{\text{vib}} = 4172.6 \pm 0.1 \text{ K} \]

4. Determine the error in the standard entropy using the information given in problems 2. and 3. and the equation given below.

\[ S^{r}_{\text{rot,vib}} = R \left( \ln \frac{T}{2\Theta_{\text{rot}}} - \ln(1-e^{-\Theta_{\text{vib}}/T}) + \frac{\Theta_{\text{vib}} / T}{e^{\Theta_{\text{vib}}/T} - 1} \right) \quad \text{where} \quad e = \exp(1) \]