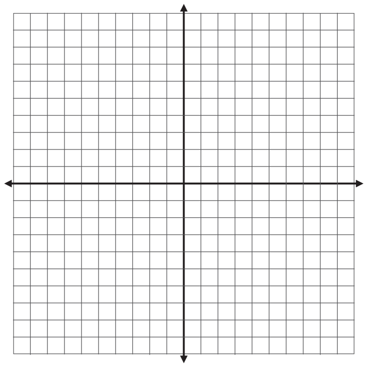
Topic 1-1 Intercepts and Lines

Definition: An intercept is a point of a graph on an axis.

For an equation Involving ordered pairs (x, y):

x-intercepts (a, 0)
y-intercepts (0, b)

where a and b are real numbers



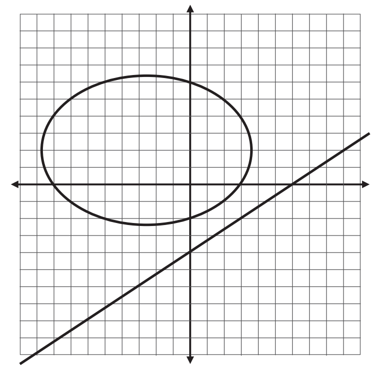
Determine the intercepts of the line and ellipse below:

Line:
x-intercept(s)

y-intercept(s)

Ellipse:
x-intercept(s)

y-intercept(s)



Linear Equations of Two Variables can be written in the form:

Ax + By = C where A, B, and C are real numbers.

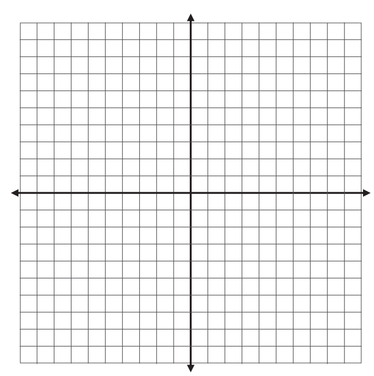
Recall that solutions to an equation are values that make the equation true.

Consider the equation $3x - 6y = 12$.

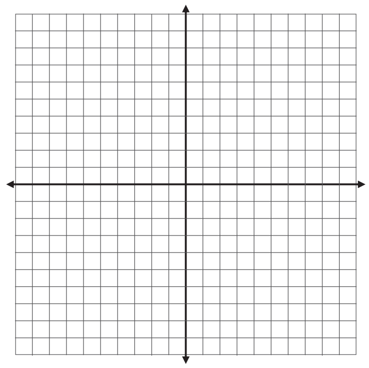
An easy way to sketch a graph of a linear equation is to find the intercepts of the equation.

To find the x-intercepts, let $y = 0$ and solve for x.
To find the y-intercepts, let $x = 0$ and solve for y.

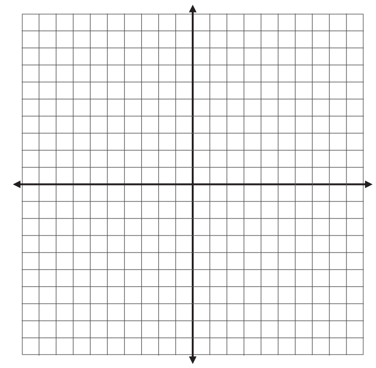
Find the intercepts of $3x - 6y = 12$ and use them to sketch the equations' graph.



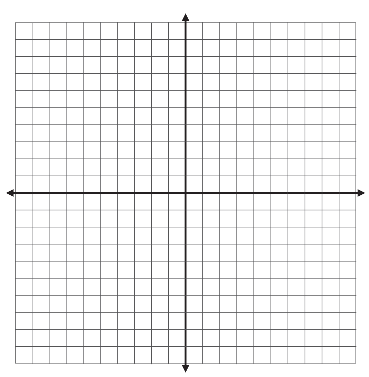
Find the intercepts of $2x - 4y = -16$ and use them to sketch the graph.



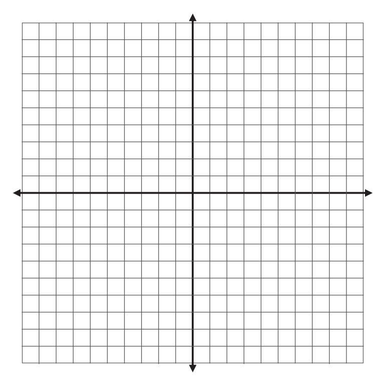
Find the intercepts of $3x + 2y = 9$ and use them to sketch the graph. Find additional points as needed.



Find the intercepts of $4x + 3y = -18$ and use them to sketch the graph. Find additional points as needed.



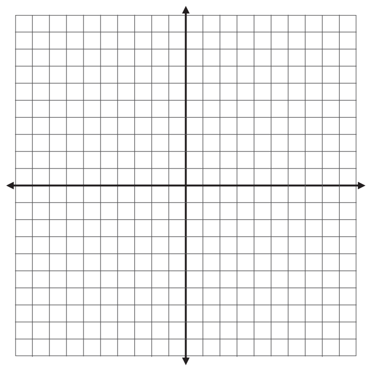
Find the intercepts of $6x + 2y = 0$ and use them to sketch the graph. Find additional points as needed.



Vertical and Horizontal Lines

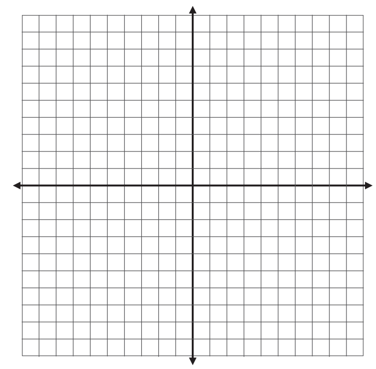
Equations of the form $x = a$ will have graphs that are vertical lines. Unless $x = 0$, it is impossible for an equation in this form to have a y-intercept.

Sketch a graph of $x = 4$.



Equations of the form $y = b$ will have graphs that are horizontal lines. Unless $y = 0$, it is impossible for an equation in this form to have an x-intercept.

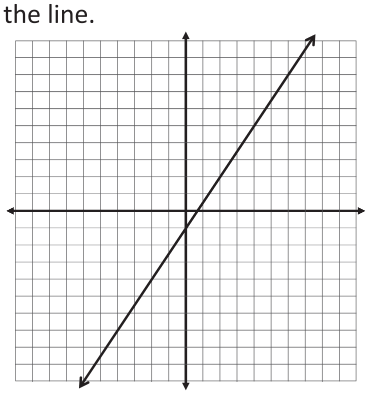
Sketch a graph of $y = 2$.



Topic 1-2 Slope and Lines

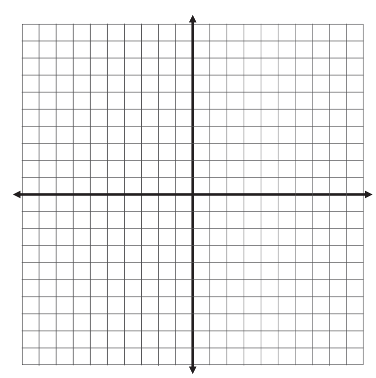
All non-vertical lines have slope.

Definition: The slope of a line is the ratio of vertical change to horizontal change over any segment of the line.

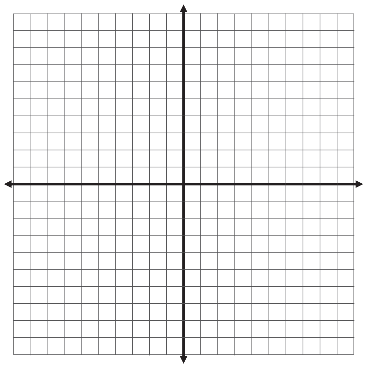


If (x_1, y_1) and (x_2, y_2) are two points on a line, then the slope m of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

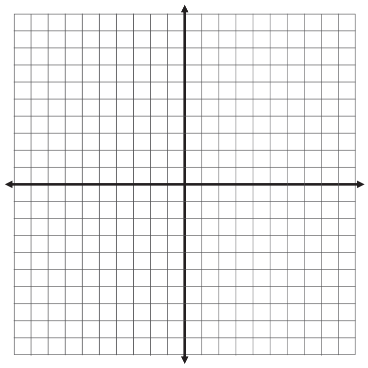
Find the slope of a line passing through (2, 5) and (6, -3). Then sketch a graph of the line.



Find the slope of a line passing through (-2, -7) and (6, -1). Then sketch a graph of the line.



Find the slope of a line passing through (5, 2) and (-1, 4). Then sketch a graph of the line.



Various facts about the slopes of lines:

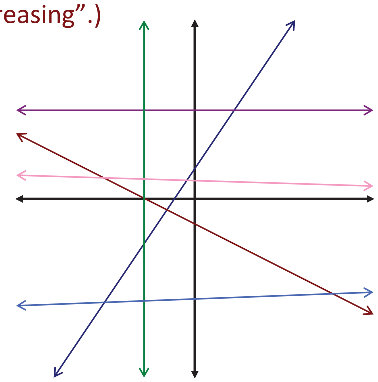
A line going up from left to right has a positive slope. (We say it is "increasing".)

A line going down from left to right has a negative slope. (We say it is "decreasing".)

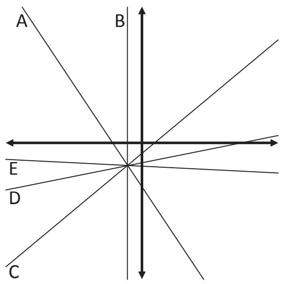
Horizontal lines ($y = b$) have a slope of 0.

Vertical lines (x = a) do not have a slope (or we might say the slope is undefined.)

The closer a line is to horizontal, the nearer its slope is to zero.



Answer each of the questions below using the graph involving lines A through E.



Which line(s) have slopes with positive values?

Which line has the slope with the least value?

Which line has a slope closest to zero?

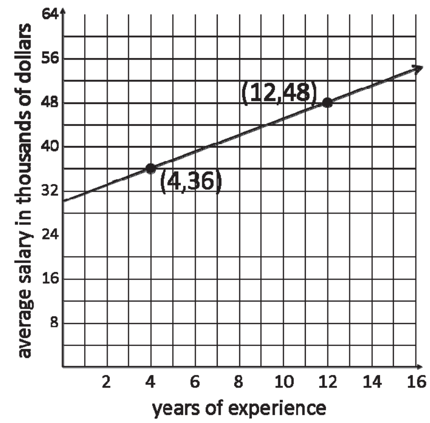
Which line appears to have no slope?

Interpreting Slope

When the meaning of the variables is known, it is possible to establish an interpretation of the value of the slope.

For instance, if x = time in hours and y = distance in miles, the difference in the values of y on top give you a change in the distance and the difference in the values of x on bottom give you a change in time. The resulting value of the slope would represent a number of miles per hour as a rate of change.

The graph given below represents the average earnings in a given profession. Interpret the slope of the linear model.



Topic 1-3 Slope-Intercept Form of Linear Equations

The equation of a line can be expressed using its slope and its y-intercept.

For a given line, if m is the slope of the line and b is its y-intercept $(0, b)$, then an equation of the line is $y = mx + b$.

Identify the slope and y-intercept of the line $y = 3x - 4$.

If a line is not expressed in slope-intercept form, it is possible to rewrite the line so that it is this form. The slope and y-intercept should be easy to find for a line in slope-intercept form.

Write each equation below in slope-intercept form and then identify the slope and y-intercept.

$2x + 3y = 9$

$2x - 6y = 5$

Reversing the process, if we know the slope and the *y*-intercept of a line, we should be able to write the equation of the line in slope-intercept form.

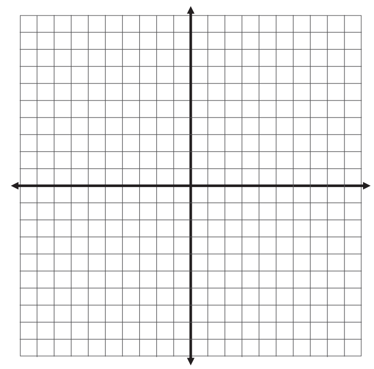
Write an equation in slope-intercept form when given the slope and *y*-intercept.

Slope: 2 *y*-intercept: (0, –3)

Slope: $-\frac{1}{2}$ *y*-intercept: (0, 4)

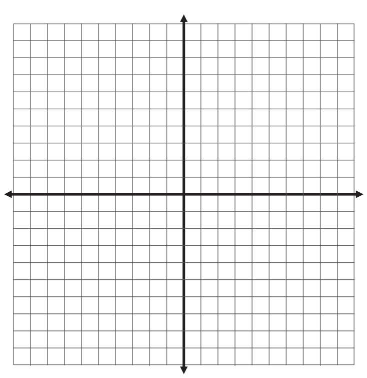
Sketching the graph of a line in slope-intercept form should be relatively easy as the *y*-intercept gives one point on the line and then the slope allows for finding additional points using the definition of slope. For this class, I expect a minimum of three points to be sketched when using slope to graph.

Sketch the graph of the line $y = 3x - 4$.



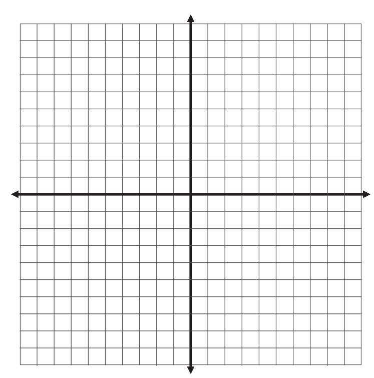
Since we know how to rewrite equations into slope-intercept form, any equation that has a slope can be graphed from this form.

Sketch the graph of the line $x - 4y = 8$.



One advantage to sketching graphs using the slope-intercept form is when the *x*-intercept of the line is not an integer but the *y*-intercept is.

Sketch the graph of the line $2x + 3y = 9$.



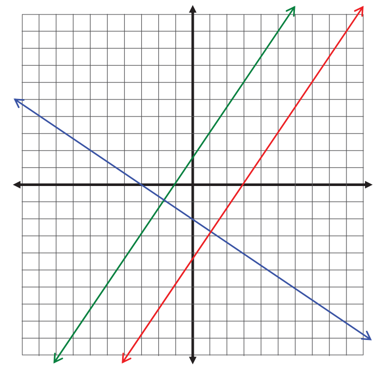
Parallel & Perpendicular Lines

Two lines l_1 and l_2 with corresponding slopes m_1 and m_2 are parallel if $m_1 = m_2$.

Two lines l_1 and l_2 with corresponding slopes m_1 and m_2 are perpendicular if $m_1 \cdot m_2 = -1$.

Vertical lines are also parallel to each other.
Horizontal lines are perpendicular to vertical lines.

Visual reference of slope with respect to parallel and perpendicular lines:



Find the slopes of lines parallel & perpendicular to a line passing through (3, 7) and (6, -1).

Are these equations parallel or perpendicular (or neither) to each other?

$$2x - 6y = 12$$
$$y = 3x - 1$$

Topic 1-4 More Forms of Linear Equations

The equation of a line can be expressed using its slope and any point on the line.

For a given line, if m is the slope of the line and (x, y) is a point on the line, then an equation of the line is $y - y_1 = m(x - x_1)$.

Notice how point-slope form flows from the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1} \Rightarrow y - y_1 = m(x - x_1)$.

Find an equation of a line with slope $-\frac{1}{2}$ passing through (2, - 3).

Forms of Linear Equations

Standard Form $Ax + By = C$

Slope-Intercept Form $y = mx + b$

Point-Slope Form $y - y_1 = m(x - x_1)$

Modified Point-Slope Form $y = m(x - x_1) + y_1$

Rarely is point-slope form used to present the equation of a line. Depending on context and formality one of the other forms is more often used.

Note: In standard form, A, B, and C are integers.

Find an equation of a line with slope $\frac{2}{3}$ passing through (-6, 1).
Give the equation in slope-intercept form.

Find an equation of a line with slope $-\frac{3}{4}$ passing through (2, 5).
Give the equation in standard form.

Find the equation of a line passing through (-3, 5) and (1, 13).
Give the equation in slope-intercept form.

Find the equation of a line passing through (-1, 6) and (3, 4).
Give the equation in standard form.

Visual relationship between horizontal & vertical lines, their slopes (or lack thereof), and their equations:

Zero slope

Horizontal Line — Equation: y = b

No slope

Vertical Line — Equation: x = a

Write the equation of a line with zero slope passing through (2, -6).

Write the equation of a vertical line passing through (3, -5).

Deep Fried Restaurant has learned that, by pricing their value pack at \$16, daily sales will be 3000 packs. Raising the price to \$20 will cause daily sales to fall to 2000 packs.

A: Express the relationship between prices and daily sales as ordered pairs. Let *x* represent price and *y* represent the daily sales of value packs in dollars.

B: Find the slope of a line passing through the ordered pairs found in part A. Interpret this slope.

C: Assume that the relationship between price and the number of value packs sold is linear. Write an equation in slope-intercept form describing the relationship.

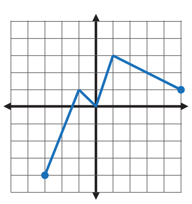
D: Estimate the number of value packs sold if the price is changed to \$18.50.

Topic 1-5 Relations and Functions

Relations are sets of ordered pairs.

Are the following things relations:

$\{(1,5),\ (4,-1),\ (-3,8),\ (2,0)\}$



$2x - 3y = 6$

Definition: The domain of a relation is the set of first values in a relation

Definition: The range of a relation is the set of second values in a relation

Definition: A function is a relation where each first value is paired with at most one second value as assigned by a rule.

Are the relations below functions?

$\{(1,5),\ (4,-1),\ (-3,8),\ (2,0)\}$

$\{(1,5),\ (4,-1),\ (-3,8),\ (2,0),\ (0,3),\ (-3,8)\}$

$\{(1,5),\ (4,-1),\ (-3,8),\ (2,0),\ (4,7),\ (1,5)\}$

$\{(1,5),\ (4,-1),\ (-3,8),\ (2,0),\ (5,-1),\ (-5,8)\}$

To determine if a graph is a function, apply the Vertical Line Test.

Vertical Line Test: for a graph to represent a function, any vertical line passing through the graph can intersect the graph at most once.

For an equation to be a function, it must be possible to uniquely write the equation in the form:

$y = \ldots$

All lines except vertical lines are functions.

Function Notation and Evaluating Functions

$f(x) = 2x - 5$

The first variable of a function is the independent variable and is expressed inside the parentheses on the left-side of the equation.

The name of a function is case-specific and is expressed prior to the parentheses on the left-side of the equation.

The second variable of a function is the dependent variable and is expressed by combining the name of the function and the first variable.

The rule of a function is the right-side of the equation and states how the value of the second variable can be found from the value of the first variable.

Identify the first and second variable of the function, the name of the function, and evaluate the function for the indicated values of the first variable.

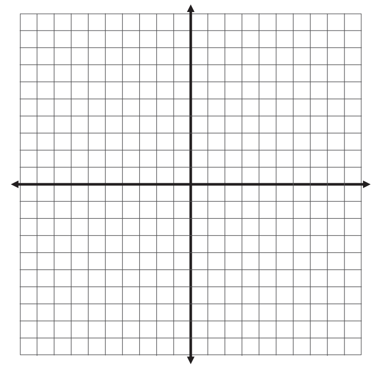
$g(x) = 3x - 2$ Find $f(4)$ and $f(-1)$

Identify the first and second variable of the function, the name of the function, and evaluate the function for the indicated values of the first variable.

$H(t) = t^2 - 2$ Find $H(1)$, $H(0)$, $H(-3)$

Observe that a function which has been evaluated corresponds closely to the ordered pair it defines:

If $h(2) = 5$, then the point (2, 5) is on the graph of h.



Topic 1-6 Linear Functions

As noted in the previous topic, all lines except vertical lines are functions.

An equation in slope-intercept form (or less frequently in modified point-slope form) implies a function. Thus linear functions are simply linear equations in slope-intercept form which are explicitly defined as functions.

Definition: A linear function is a function of the form $f(x) = \mathrm{m}x + b$ where m is the slope of the line sketched by the function and b is the y-intercept $(0, b)$ that the line passes through.

$y = \mathrm{m}x + b$: Linear equation, implied linear function
$f(x) = \mathrm{m}x + b$: Linear equation, explicit linear function

Write a linear function which has a slope of 4 and a y-intercept of (0, −2).

Write a linear function which has a slope of $-\frac{3}{2}$ and passes through (4, −1).

Like working with linear equations, if two points of a linear function are known, then the slope can be found and one of the points plus the slope can be used to find the linear function.

Write a linear function which passes through (2, 6) and (6, −4).

Write a linear function which is horizontal and passes through (6, −4).

Write a linear function which passes through (2, 6) and is parallel to $f(x) = 2x - 5$.

Write a linear function which passes through (6, −4) and is perpendicular to $f(x) = 2x - 5$.

Sketching the graph of a linear function is also just like sketching the graph of a linear equation: use the slope for rise over run and the *y*-intercept as a starting point.

Sketch the graph of $f(x) = 2x - 5$.