Unit 5 Graphing Trigonmetric Functions

This is a BASIC CALCULATORS ONLY unit.

Know the meanings and uses of these terms:

Period *(the value)*
Period *(the interval)*
Amplitude

Review the meanings and uses of these terms:

Domain of a function
Range of a function
Translation of a graph
Reflection of a graph
Dilation of a graph
Asymptote

Periodic Functions

Trigonometric functions are periodic.

Definition: A function f is periodic if there exists a positive number p such that $f(t + p) = f(t)$ for every t.

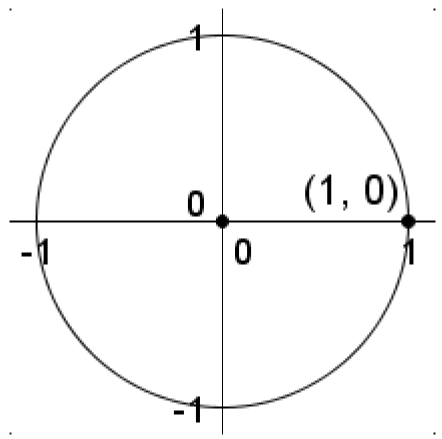
If f has period p, then the graph of f on any interval of length p is called one complete period of f.

Since sine and cosine are defined by the terminal point of t and the addition of $2n\pi$ (n is an integer) to t is coterminal to t, then periodic behavior of of sine and cosine must occur over an interval of 2π.

$\sin(t + 2\pi) = \sin t$ $\cos(t + 2\pi) = \cos t$

Derivation of graph of sin t

Recall that $\sin t = y$, where y is the y-value of the terminal point determined by t.



Recall the domain of sine is $\mathbb{R}$.

Observe that the maximum possible value of sine

is

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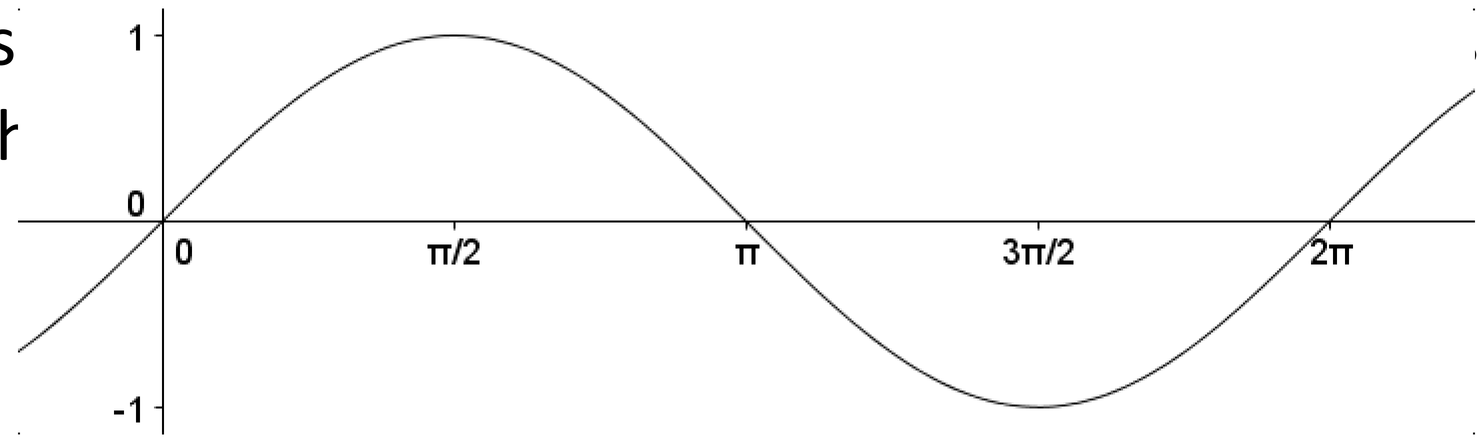
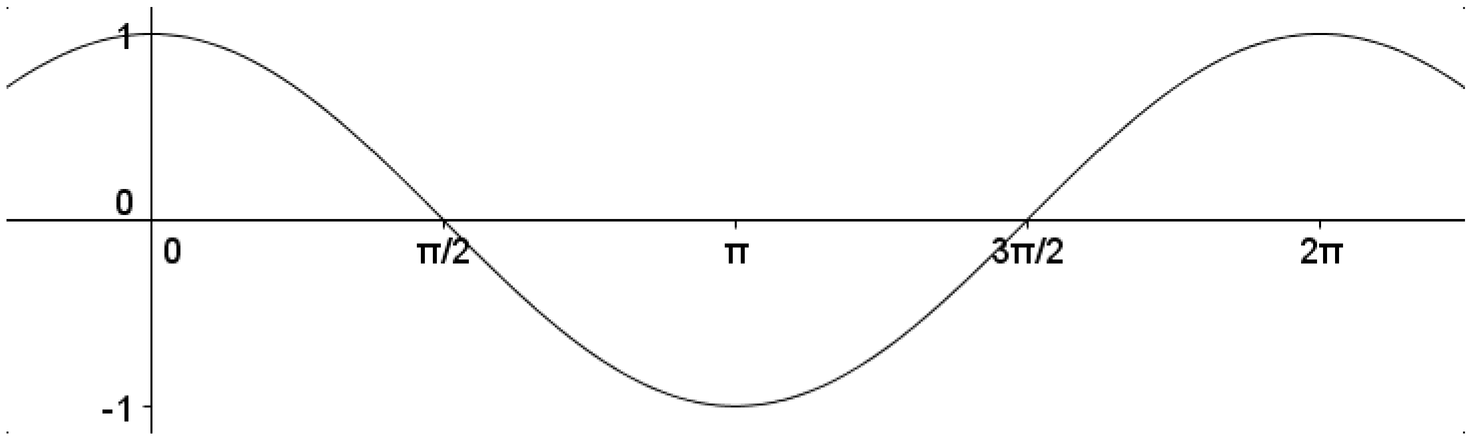


Presentation of graph of cos *t*

Recall that cos *t* = *x*, where *x* is the *x*-value of the terminal point determined by *t*.



Cosine appears as shifted representation of sine.

Like sine, cosine has a domain of $\mathbb{R}$.

Also, like sine, cosine has a range of [–1, 1].

Observe that the most basic complete period of sine or cosine is the interval $[0, 2\pi]$.

Transformations of Trigonometric Functions

$y = a \sin k(x - b) + c$ $\qquad$ $y = a \cos k(x - b) + c$

a: If $|a| > 1$, sin/cos is stretched away from the *x*-axis

If $|a| < 1$, sin/cos is compressed toward the *x*-axis

If *a* is negative, sin/cos is reflected about the *x*-axis

k: If $|k| < 1$, sin/cos is stretched away from the *y*-axis

If $|k| > 1$, sin/cos is compressed toward the *x*-axis

b: If *b* is positive, sin/cos is shifted to the right $(x - \#)$

If *b* is negative, sin/cos is shifted to the left $(x + \#)$

c: If *c* is positive, sin/cos is shifted upward

If *c* is negative, sin/cos is shifted downward

Properties of a sine/cosine graph:

Dilations with respect to the *y*-axis create changes in the **period** of a trigonometric function.

Dilations with respect to the *x*-axis create changes in the **amplitude** of a trigonometric function.

Translations horizontally create a **phase shift** compared to the basic trigonometric function.

Translations vertically create a **vertical shift** compared to the basic trigonometric function.

Negations effect the location of peaks and valleys in a trigonometric function.

period = $\dfrac{2\pi}{k}$ amplitude = $|a|$ phase shift = b

Expectations for Trigonometric Graphs, pt 1:

For sine and cosine functions, these are my expectations:

1. Identify the period, amplitude, & phase shift of the sine or cosine graph.
2. Determine the domain of the primary complete period.

   For sine and cosine functions, the primary complete period will be over $\left[b,\ \frac{2\pi}{k}+b\right]$.
3. Determine the range of the graph.

   For sine and cosine functions, the range will be $\left[-|a|+c,\ |a|+c\right]$.
4. Mark and label the endpoints of the domain on the *x*-axis.
5. Mark and label the midpoint of the domain and the midpoints between an endpoint and a midpoint (which I refer to as "quarterpoints").
6. Mark and label the endpoints of the range and the midpoint of the range on the *y*-axis.
7. Evaluate the function at the five values marked on the *x*-axis. If everything has been done correctly, the value of the function at these *x*-values should correspond to one of the *y*-values marked on the *y*-axis.

Sketch a graph of the trigonometric function and identify its properties.

Ex. 1: $y = 3\sin 2x$

Period to be Graphed: [____ , ____]

Range: [____ , ____]

Period = ____

Amplitude = ____

Phase Shift = ____

Sketch a graph of the trigonometric function and identify its properties.

Ex. 2: $y = 2\cos\frac{x}{3}$

Period to be Graphed: [____ , ____]

Range: [____ , ____]

Period = ____

Amplitude = ____

Phase Shift = ____

Sketch a graph of the trigonometric function and identify its properties.

Ex. 3: $y = 2\sin x - 1$

Period to be Graphed: [____ , ____]

Range: [____ , ____]

Period = ____

Amplitude = ____

Phase Shift = ____

Sketch a graph of the trigonometric function and identify its properties.

Period to be Graphed: [____ , ____]

Ex. 4: $y = \frac{1}{2}\cos\left(x - \frac{\pi}{3}\right)$

Range: [____ , ____]

Period = ____

Amplitude = ____

Phase Shift = ____

Sketch a graph of the trigonometric function and identify its properties.

Ex. 5: $y = -4\sin\left[\frac{1}{2}\left(x + \frac{\pi}{4}\right)\right]$

Period to be Graphed: [____ , ____]

Range: [____ , ____]

Period = ____

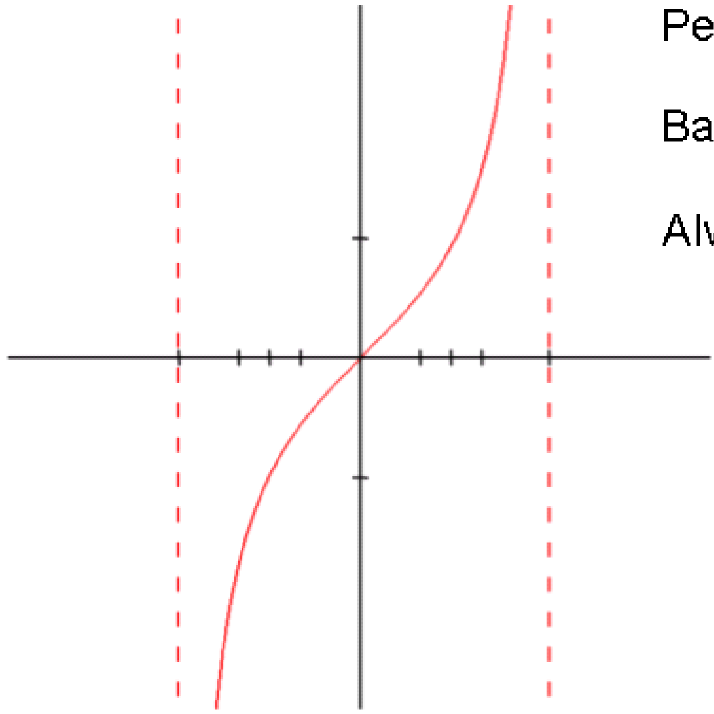
Amplitude = ____

Phase Shift = ____

Basic Graphs of Tangent and Cotangent Functions

Graph of Tangent:

x	$\tan x$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3}$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	undefined



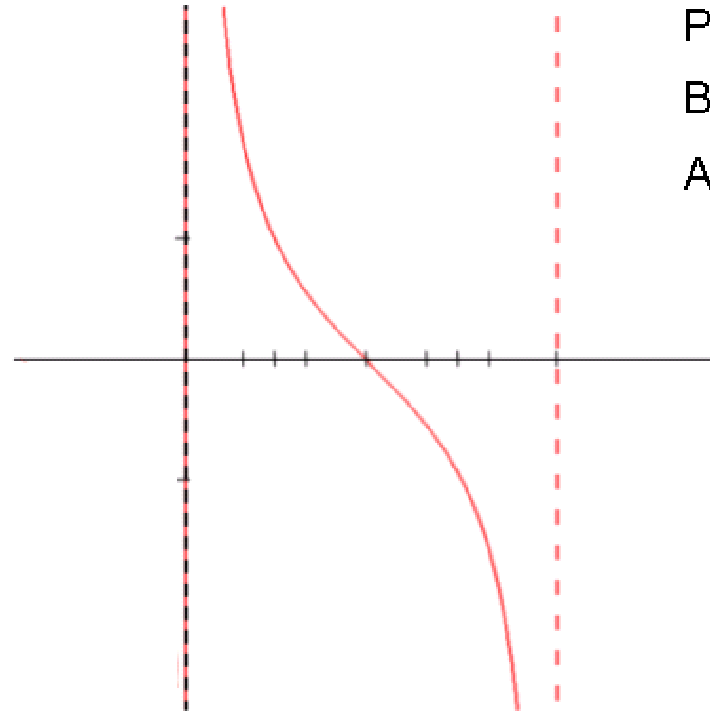
Period Length $= \pi$

Basic Period: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Always Increasing

Graph of Cotangent:

x	$\cot x$
0	undefined
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{\sqrt{3}}{3}$
$\frac{3\pi}{4}$	-1
$\frac{5\pi}{6}$	$-\sqrt{3}$
π	undefined



Period Length $= \pi$

Basic Period: $(0, \pi)$

Always Decreasing

General Form: $y = a \tan k(x - b) + c$

Period $= \dfrac{\pi}{k}$

Domain of Primary Period: $\left(-\frac{\pi}{2k} + b, \frac{\pi}{2k} + b\right)$

Period to be Graphed: $\left[-\frac{\pi}{2k} + b, \frac{\pi}{2k} + b\right]$

Range: $(-\infty, \infty)$

General Form: $y = a \cot k(x - b) + c$

Period $= \dfrac{\pi}{k}$

Domain of Primary Period: $\left(b, \frac{\pi}{k} + b\right)$

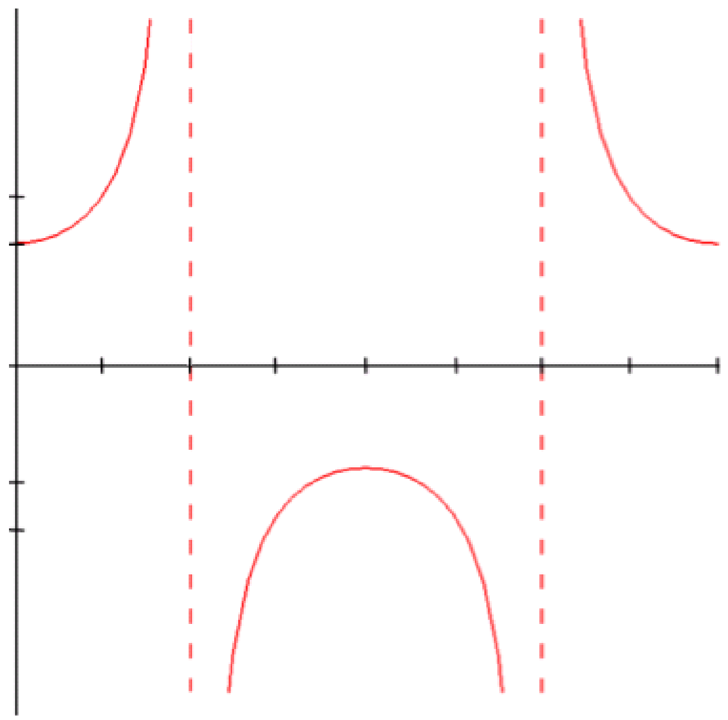
Period to be Graphed: $\left[b, \frac{\pi}{k} + b\right]$

Range: $(-\infty, \infty)$

Basic Graphs of Secant and Cosecant Functions

Graph of Secant:

x	sec x
0	1
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{2}$	undefined
$\frac{3\pi}{4}$	$-\sqrt{2}$
π	-1
$\frac{5\pi}{4}$	$-\sqrt{2}$
$\frac{3\pi}{2}$	undefined
$\frac{7\pi}{4}$	$\sqrt{2}$
2π	1



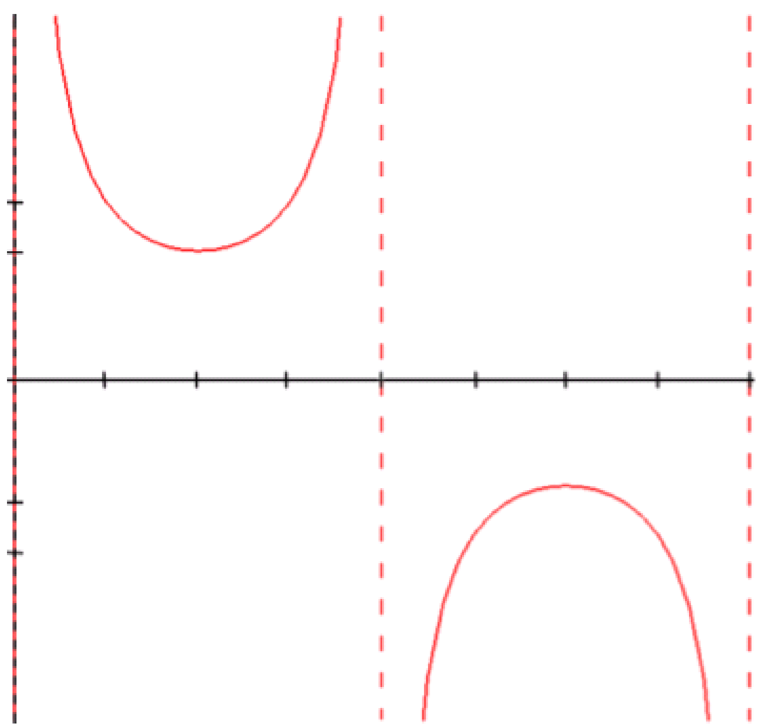
Period Length = 2π

Basic Period:

$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right]$

Graph of Cosecant:

x	csc x
0	undefined
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{2}$	1
$\frac{3\pi}{4}$	$\sqrt{2}$
π	undefined
$\frac{5\pi}{4}$	$-\sqrt{2}$
$\frac{3\pi}{2}$	-1
$\frac{7\pi}{4}$	$-\sqrt{2}$
2π	undefined



Period Length = 2π

Basic Period:

$(0,\pi)\cup(\pi,2\pi)$

General Form: $y = a \sec k(x - b) + c$

General Form: $y = a \csc k(x - b) + c$

$$\text{Period} = \frac{2\pi}{k}$$

Period to be Graphed: $\left[b,\ \frac{2\pi}{k}+b\right]$

Range: $\left(-\infty,-|a|\right]\cup\left[|a|,\infty\right)$

For the remaining functions, these are my expectations:

1. Identify the period & phase shift of the trigonometric functions. Also note any vertical dilations or translations.
2. Mark and label the endpoints of the domain on the *x*-axis.
3. Mark and label the midpoint and the "quarterpoints".
4. Mark and label three/two points on the *y*-axis:
   $y=|a|+c,\quad y=-|a|+c,\quad y=c$ (third only for tan/cot)
5. Evaluate the function at the five values marked on the *x*-axis. The value of the function at each *x*-value should either be a value on the *y*-axis or undefined. Asymptotes will exist where the function is undefined.

Sketch a graph of the trigonometric function and identify its properties.

Ex. 1: $y = 3\tan 2x$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 2: $y = 4\cot x$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 3: $y = 2\tan\frac{x}{4}$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_ , \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 4: $y = \cot 3x$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_ , \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 5: $y = 4\csc 2x$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 6: $y = \frac{1}{2}\sec 3x$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 7: $y = \frac{1}{2}\csc\frac{x}{2}$

Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

Sketch a graph of the trigonometric function and identify its properties.

Ex. 8: $y = -2\sec\frac{x}{5}$

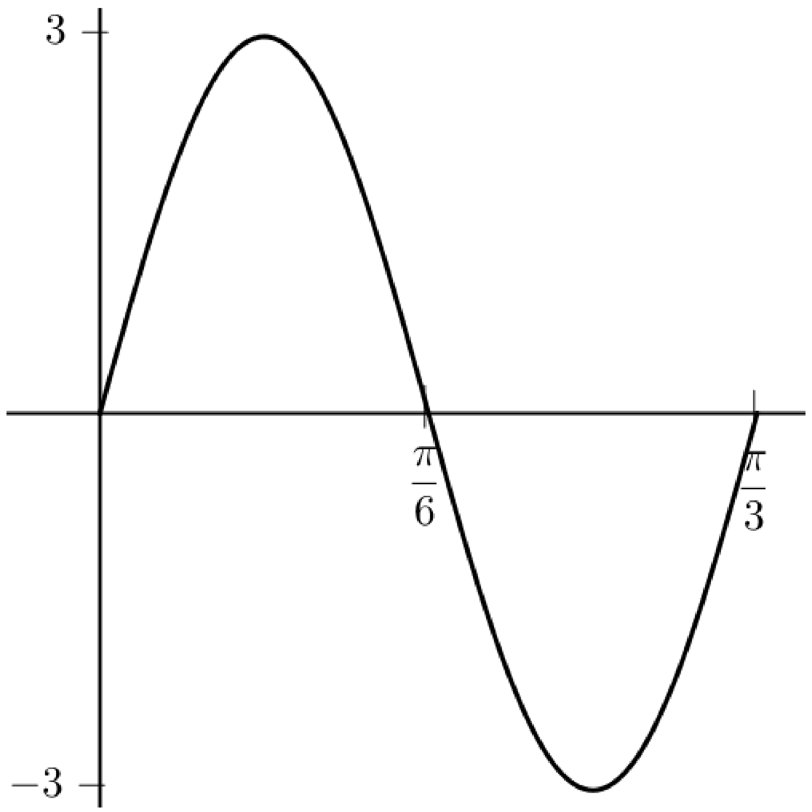
Period = ____

Period to be Graphed: $\left[\ \_\_\_\_\ ,\ \_\_\_\_\ \right]$

A Second Look at the Sine and Cosine Graphs

The graph of a complete period of sine is shown below. Find the amplitude, period, and phase shift.

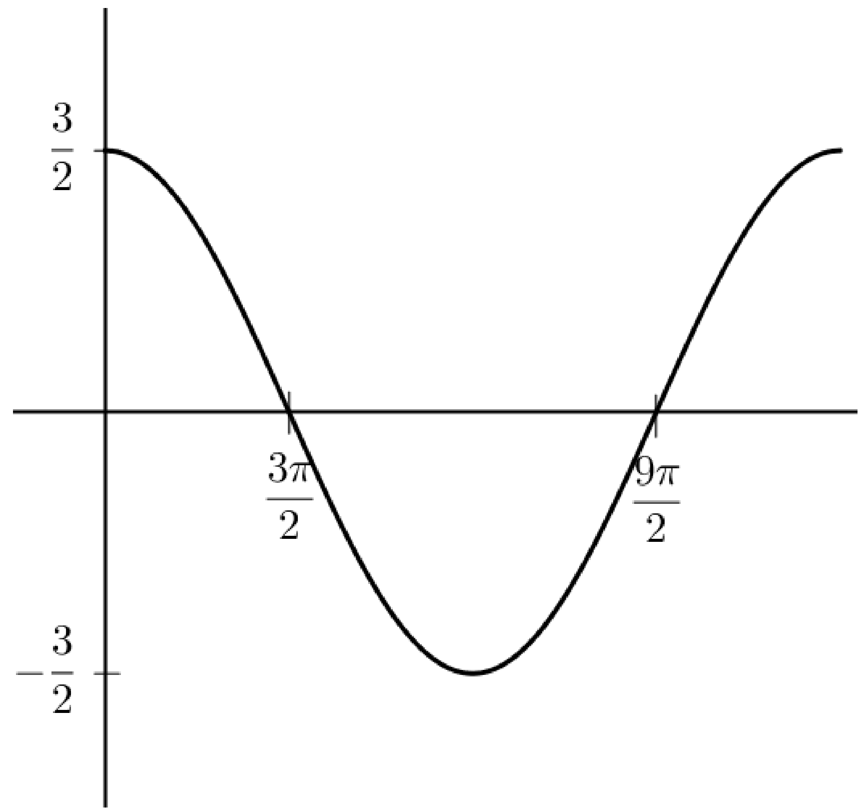
Ex. 1:



Identify the equation $y = a\sin\left(k\left(x-b\right)\right)$ that is represented by the curve.

The graph of a complete period of cosine is shown below. Find the amplitude, period, and phase shift.

Ex. 2:



Identify the equation $y = a\cos\left(k\left(x-b\right)\right)$ that is represented by the curve.

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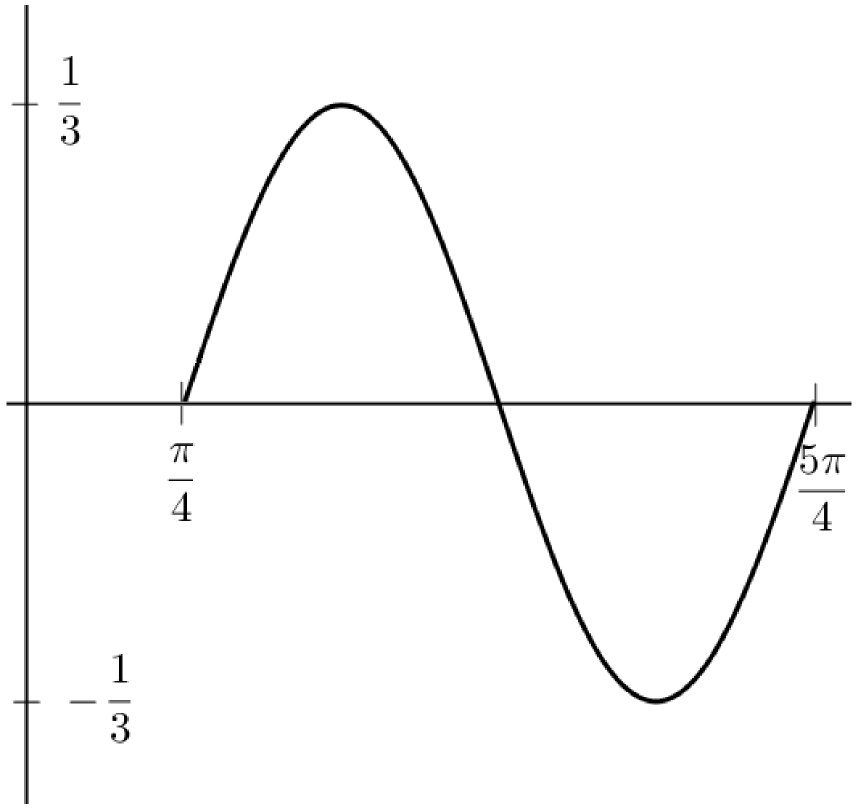
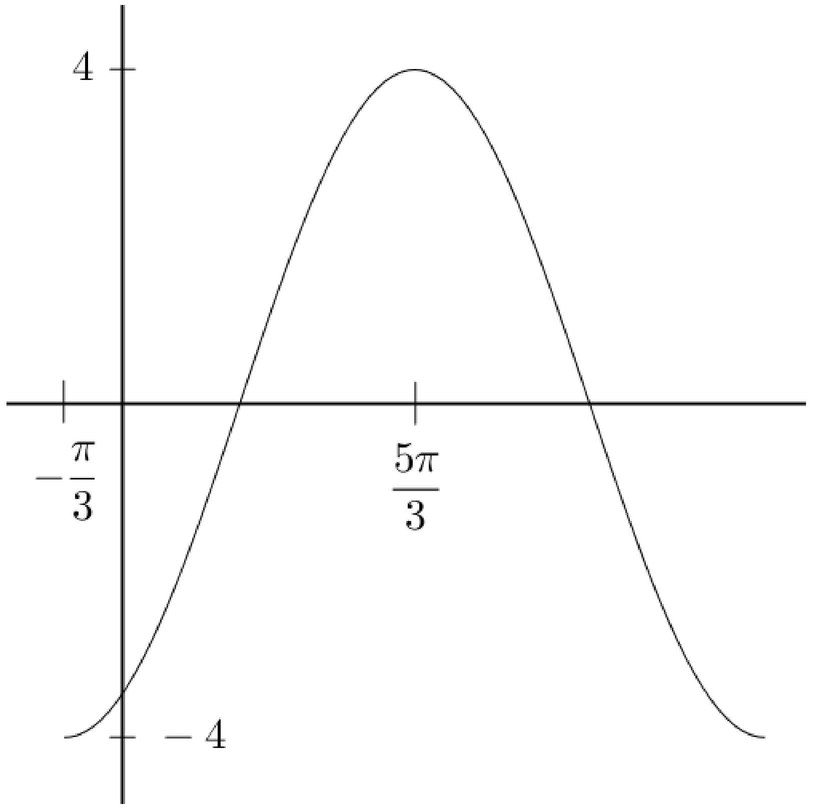


Identify the equation $y = a\sin\left(k\left(x-b\right)\right)$ that is represented by the curve.

The graph of a complete period of cosine is shown below. Find the amplitude, period, and phase shift.

Ex. 4:



Identify the equation $y = a\cos\left(k\left(x-b\right)\right)$ that is represented by the curve.

Simple Harmonic Motion

Many objects in nature and science, such as springs, strings, and waves for sound and light, can be modeled by sine and cosine graph.

Definition: An object is in simple harmonic motion if its displacement y as an object of time either can be defined by the equation $y = a \sin \omega t$ (when the displacement is zero at time 0) or the equation $y = a \cos \omega t$ (when the displacement is maximized at time 0).

The amplitude of displacement is $|a|$.
The period of one cycle is $2\pi/\omega$.

The frequency is $\omega/2\pi$.

Definition: Frequency is the number of cycles occurring per unit of time.

The given function models the displacement of an object moving in simple harmonic motion. Find the amplitude, period, and frequency of the motion, assuming time is in seconds.

Ex. 1: $y = 4\sin 6t$

Ex. 2: $y = 2\cos \frac{1}{4}t$

Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is zero at time $t = 0$.

Ex. 1: Amplitude 20 in, Period 10 sec

Ex. 2: Amplitude 1.5 m, Frequency 90 Hz

Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is maximized at time $t = 0$.

Ex. 1: Amplitude 100 ft, Period 2 min

Ex. 2: Amplitude 4.2 cm, Frequency 220 Hz