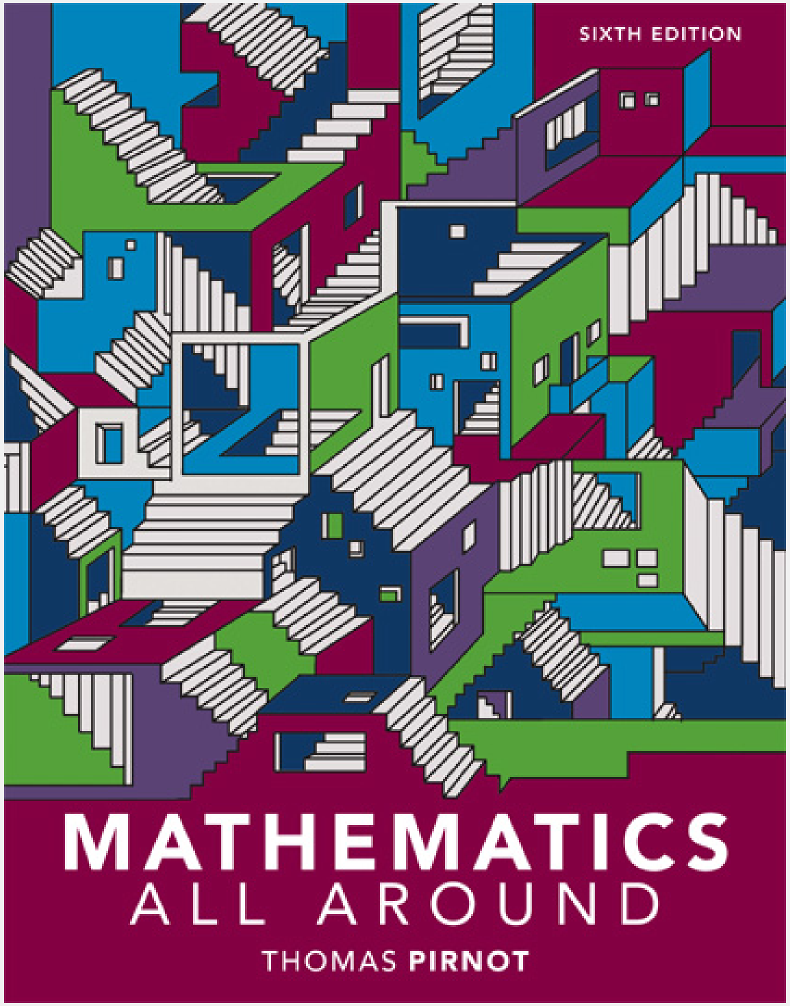
7.3

Quadratic Models



7.3 Modeling with Quadratic Equations

- Use the quadratic formula to solve equations.
- Graph a quadratic equation.
- Use a quadratic equation to model data.

Quadratic Equations

An equation of the form

$$y = ax^2 + bx + c \qquad a \neq 0$$

is called a **quadratic equation**.

As with linear equations in two variables, solutions to quadratic equations are ordered pairs of numbers.

Example: Verifying Solutions for Quadratic Equations in Two Variables

Determine whether (1, 2) is a solution for the quadratic equation $y = 2x^2 - 3x + 5$.

Solution

Substitute 1 for x and 2 for y.

$y = 2x^2 - 3x + 5$

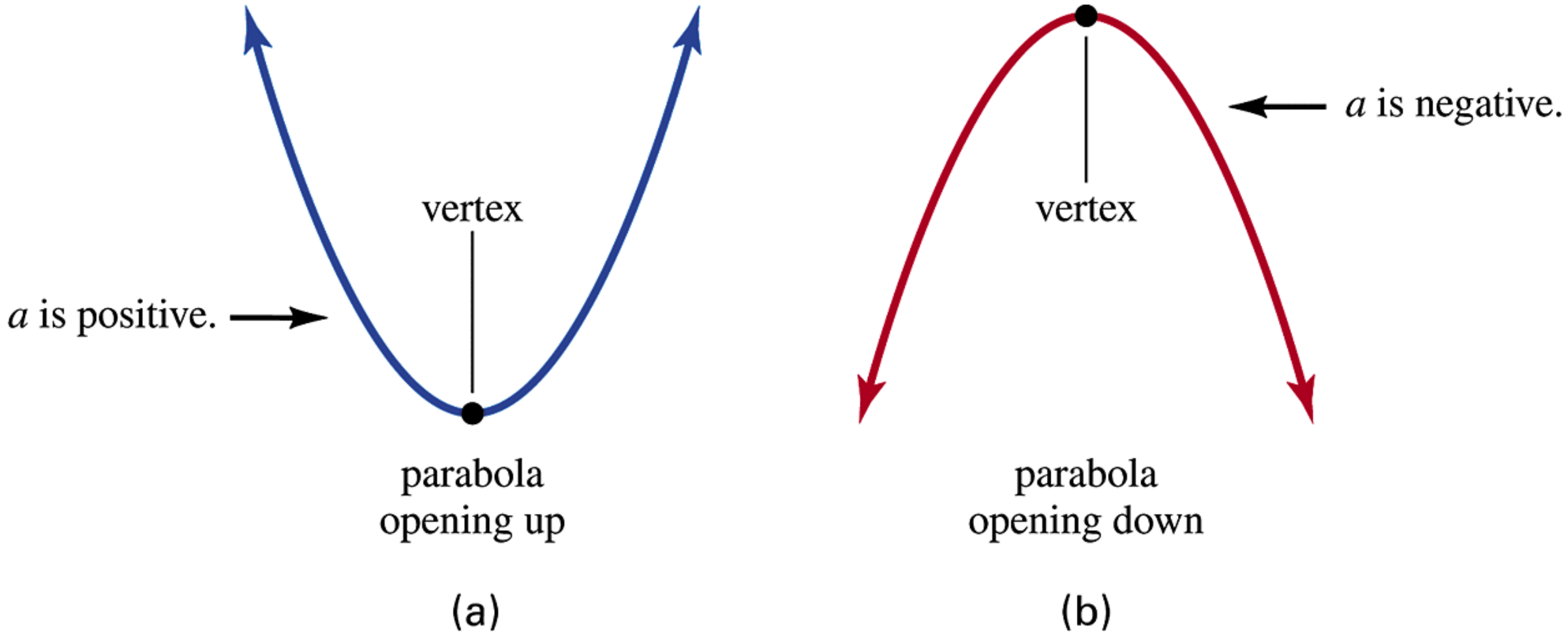
$2 = 2(1) - 3(1) + 5$

$2 = 4$ FALSE

(1, 2) is not a solution.

Quadratic Equations

The graph of a quadratic equation is a parabola.



Quadratic Equations

The **vertex** of the graph of the quadratic equation $y = ax^2 + bx + c$ occurs when

$$x = \frac{-b}{2a}.$$

Example: Finding the Vertex of a Parabola

Find the vertex of the graph of $y = 2x^2 - 4x + 5.$

Solution

$a = 2,\ b = -4,\ c = 5$

$$x = \frac{-b}{2a}$$

$$= \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$$

Example: Finding the Vertex of a Parabola (cont)

Substituting 1 for x, we get the y-coordinate of the vertex.

$y = 2x^2 - 4x + 5$

$y = 2(1)^2 - 4(1) + 5$

$= 2 - 4 + 5$

$= 3$

The vertex of the parabola is the point (1, 3).

Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Combined we have $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example: Using the Quadratic Formula to Solve an Equation

Use the quadratic formula to solve the equation $x^2 + 5x - 84 = 0$.

Solution

$a = 1$, $b = 5$, and $c = -84$

We substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-84)}}{2(1)}$$

Example: Using the Quadratic Formula to Solve an Equation (cont)

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-84)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 336}}{2}$$

$$x = \frac{-5 \pm \sqrt{361}}{2}$$

$$x = \frac{-5+19}{2} = 7 \text{ and } x = \frac{-5-19}{2} = -12$$

The solutions are 7 and –12.

Discriminant

- $b^2 - 4ac$ is greater than 0: There are two distinct solutions to the equation.
- $b^2 - 4ac$ is equal to 0: There is only one solution to the equation.
- $b^2 - 4ac$ is less than 0: There are no real-number solutions to this equation, since the square root of a negative number is not defined in the real-number system.

Example: Graphing a Quadratic Equation

Graph the quadratic equation $y = x^2 - 4x - 12$, which is a parabola, by doing the following:

a) Determine if the parabola is opening up or down.

b) Find the vertex of the parabola.

c) Determine the number of solutions to the equation $x^2 - 4x - 12 = 0$.

d) Find the x- and y-intercepts of the graph.

e) Draw the graph.

Example: Graphing a Quadratic Equation (cont)

Solution

a) Opens up because the coefficient for x^2 is 1.

b) Vertex

$a = 1$, $b = -4$, and $c = -12$

$$x = \frac{-b}{2a} = \frac{-(-4)}{2 \cdot 1} = \frac{4}{2} = 2$$

Find the *y*-coordinate.

$$y = 2^2 - 4(2) - 12 = 4 - 8 - 12 = -16$$

Vertex is at (2, –16).

Example: Graphing a Quadratic Equation (cont)

c) Number of solutions $x^2 - 4x - 12 = 0$ use the discriminant.

$a = 1$, $b = -4$, and $c = -12$

$b^2 - 4ac = (-4)^2 - 4(1)(-12) = 16 + 48 = 64$

The discriminant is greater than 0, there are two solutions so there will be two x-intercepts.

Example: Graphing a Quadratic Equation (cont)

d) There are two *x*-intercepts. Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 48}}{2}$$

$$= \frac{4 \pm \sqrt{64}}{2} = \frac{4 \pm 8}{2} = 6 \text{ and } -2$$

$$(-2, 0) \text{ and } (6, 0)$$

Example: Graphing a Quadratic Equation (cont)

d) y-intercept

$$y = x^2 - 4x - 12$$

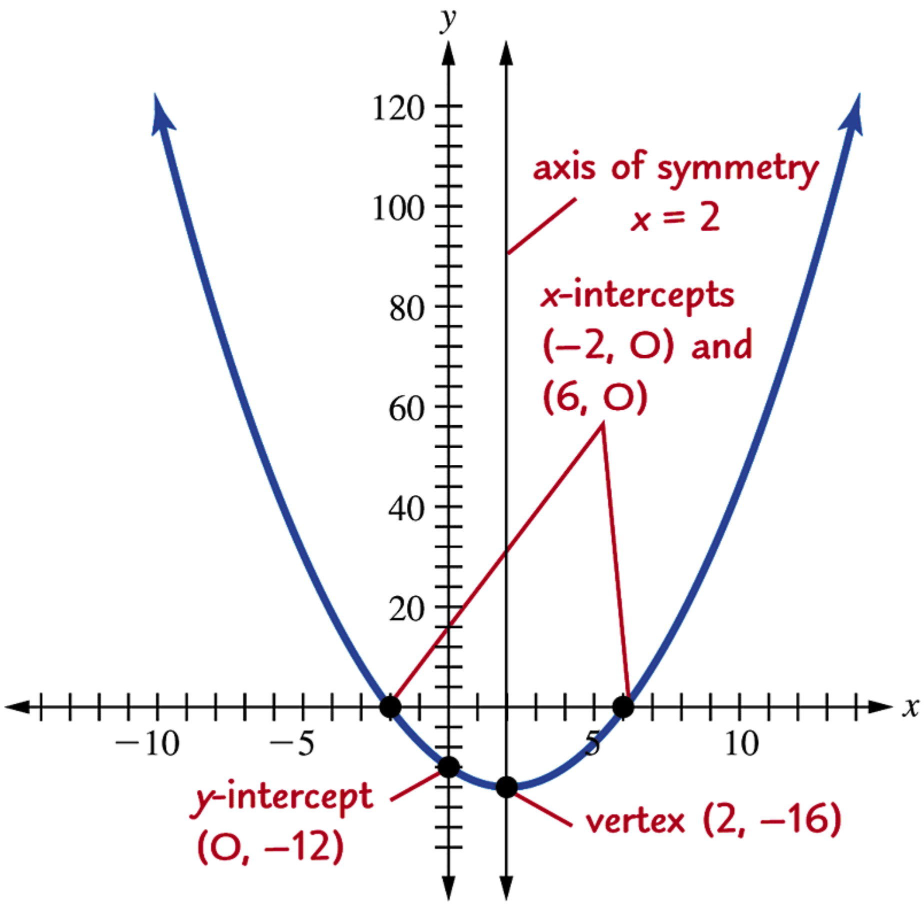
$$y = (0)^2 - 4(0) - 12$$

$$y = -12$$

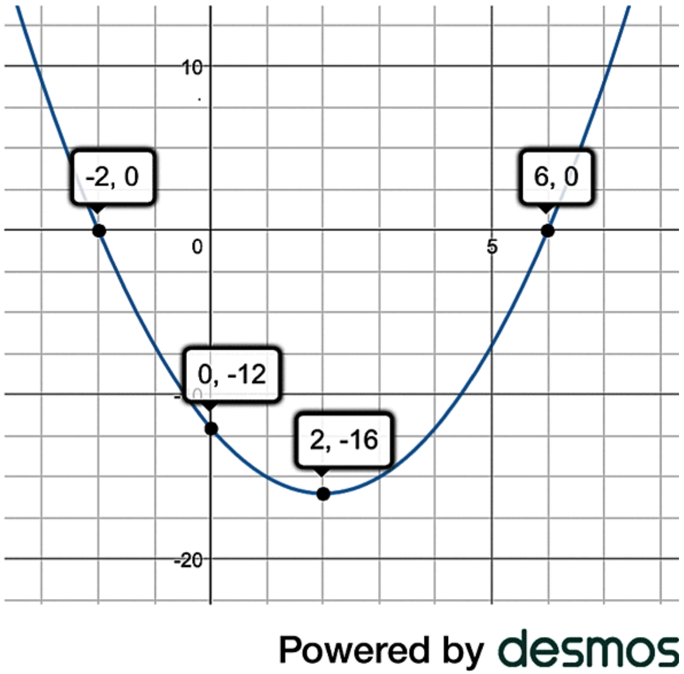
$(0, -12)$

Using the information we can now draw a reasonable graph. (see next slide)

Example: Graphing a Quadratic Equation (cont)



(a)



(b)

Modeling with Quadratic Equations

We will now apply our knowledge of quadratic equations to model building. After the release of a new video game, there are four stages in its life cycle:

- Stage 1: Sales increase rapidly. Everyone wants to play!
- Stage 2: The sales are still growing, but the increase from week to week is not as great as in the early phase.

Modeling with Quadratic Equations (cont)

- Stage 3: The product is still selling, but now each week's sales are a little lower than the week before.
- Stage 4: The market is saturated, and sales are now dropping rapidly.

Modeling with Quadratic Equations (cont)