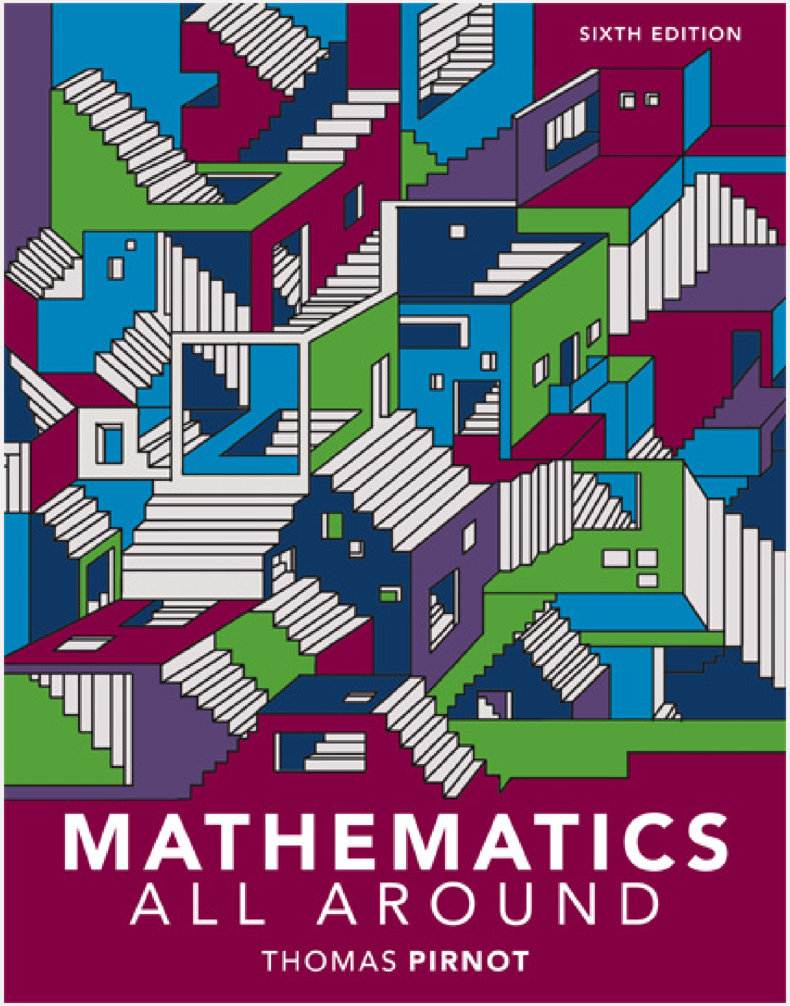
7.2

Linear Models



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7.2 Modeling with Linear Equations

- Build a linear model using a point and the slope.
- Use two points to build a linear model.
- Describe how to use the line of best fit to model real data.

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Specifying Linear Models

There are several ways that we can specify information that determines a linear equation as a model.

Method of Specifying Equation	Information Provided
Write the equation in standard form.	$3x + 2y = 6$
State the x- and y-intercepts of the graph of the equation.	x-intercept is (2, 0). y-intercept is (0, 3).
Specify the slope and y-intercept of the graph of the equation.	Slope is $-\frac{3}{2}$ and y-intercept is 3. Slope–intercept form of the equation is $y = -\frac{3}{2}x + 3$.

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Example: Using the Slope and a Point to Determine a Linear Equation

Find a linear equation of the line with slope 3 passing through the point (4, 5).

Solution

We will assume that we can write the equation of the line in slope–intercept form, $y = mx + b$. The slope of the graph of the equation is $m = 3$, so we can rewrite this equation as

$$y = 3x + b.$$

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Example: Using the Slope and a Point to Determine a Linear Equation (cont)

We need to find *b.* Because (4, 5) lies on the line, we substitute 4 for *x* and 5 for *y* to get

$$5 = 3(4) + b$$

$$5 = 12 + b$$

$$-7 = b$$

The equation is $y = 3x - 7$.

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Example: Using Two Points to Find a Linear Equation

Find an equation of the line passing through the points (4, 2) and (8, 5).

Solution

We will write the equation in slope–intercept form as $y = mx + b$, so we need to find *m* and *b*. Using points (4, 2) and (8, 5), we find the slope:

$$m = \frac{rise}{run} = \frac{5-2}{8-4} = \frac{3}{4}.$$

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Example: Using Two Points to Find a Linear Equation (cont)

Substituting for *m* we get

$$y = \frac{3}{4}x + b.$$

Because (4, 2) lies on the graph, we substitute 4 for *x* and 2 for *y* to get

$$2 = \frac{3}{4}(4) + b$$

$$2 = 3 + b$$

$$-1 = b$$

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Example: Using Two Points to Find a Linear Equation (cont)

The final equation is

$$y = \frac{3}{4}x - 1.$$

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The Line of Best Fit

- Real data is almost never perfectly linear.
- Nevertheless, if the data exhibit a *linear trend*, we can model the data with a *line of best fit*.
- The line of best fit gives the best linear approximation of the data.

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Example: Modeling Payments Made for Digital Radio Services

The figure (next slide) shows a graph of payments made to performers and copyright holders for digital performance and streaming services between 2009 and 2013, as reported by the Recording Industry Association of America. Although the points do not all lie on a straight line, we may wish to model the data with a linear model.

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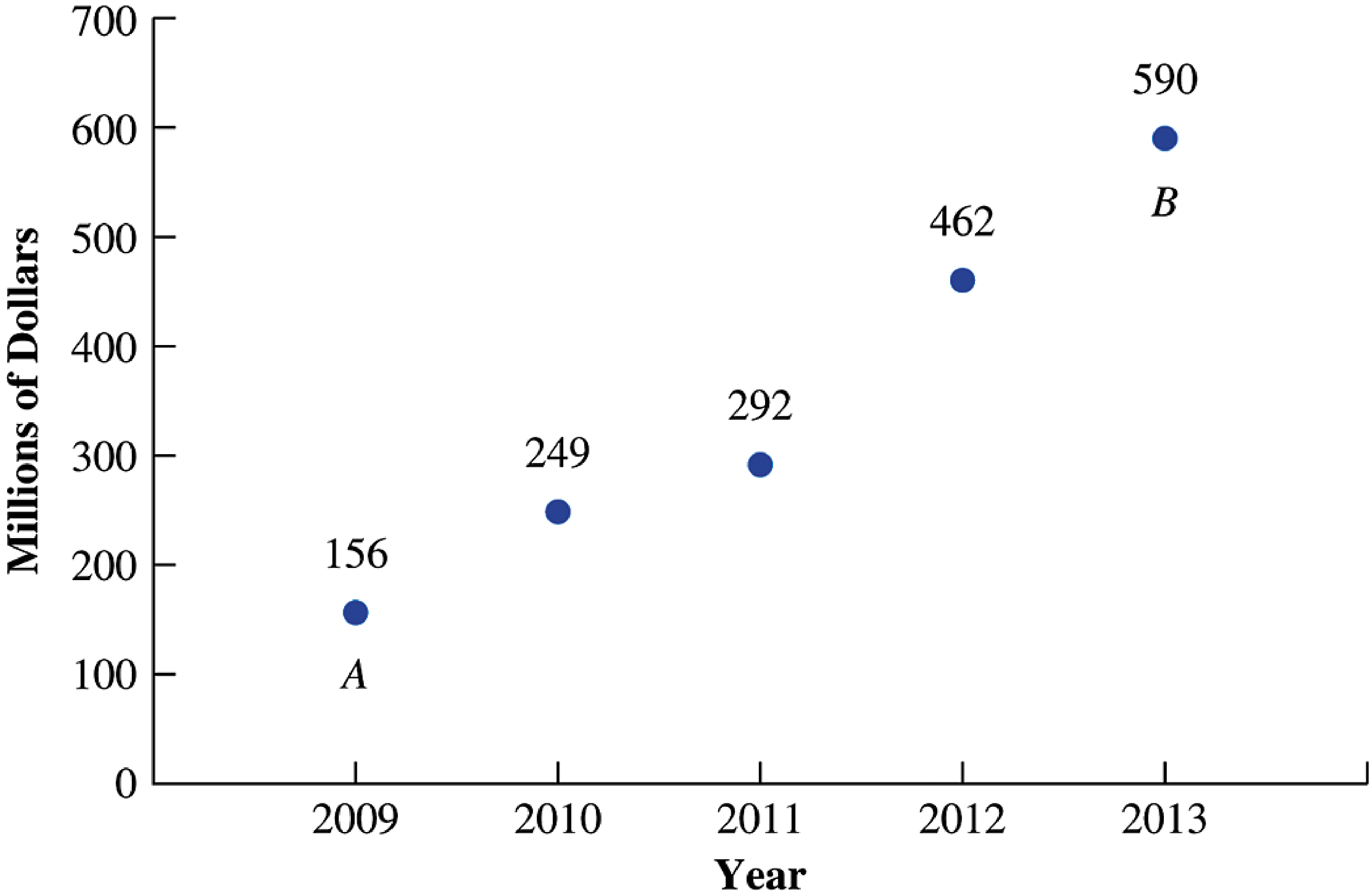
Example: Modeling Payments Made for Digital Radio Services (cont)



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Example: Modeling Payments Made for Digital Radio Services (cont)

a) Model the data by finding the equation of a line that passes through points *A* and *B*.

b) Use this model to predict payments for digital performance and streaming in 2014.

c) The actual payments for digital performance and streaming in 2014 were $773 million. Compare this to your answer in part (b) and evaluate the accuracy of your model.

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Example: Modeling Payments Made for Digital Radio Services (cont)

Solution

a) Model the data by finding the equation of a line that passes through points *A* and *B*.

Using the technique of a previous example (calling year 2009 $t = 0$, etc.), we find that an equation of the line that passes through points (0, 156) and (4, 590) is $s = 108.5t + 156$, where t is the time in years after 2009 and s is the payments for digital performance in millions of dollars (verify).

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Example: Modeling Payments Made for Digital Radio Services (cont)

b) This equation predicts that in 2014 (year 5) the payments due to digital performance will be

s = 108.5(5) + 156 = 698.5 million dollars.

c) Looking at the actual data, we see that the payments due to digital performance and streaming were $773 million. As shown in the figure on the next slide, our model's prediction is low, which suggests that while the model from 2009 to 2013 looked fine, we should be careful about using our model for long-term predictions.

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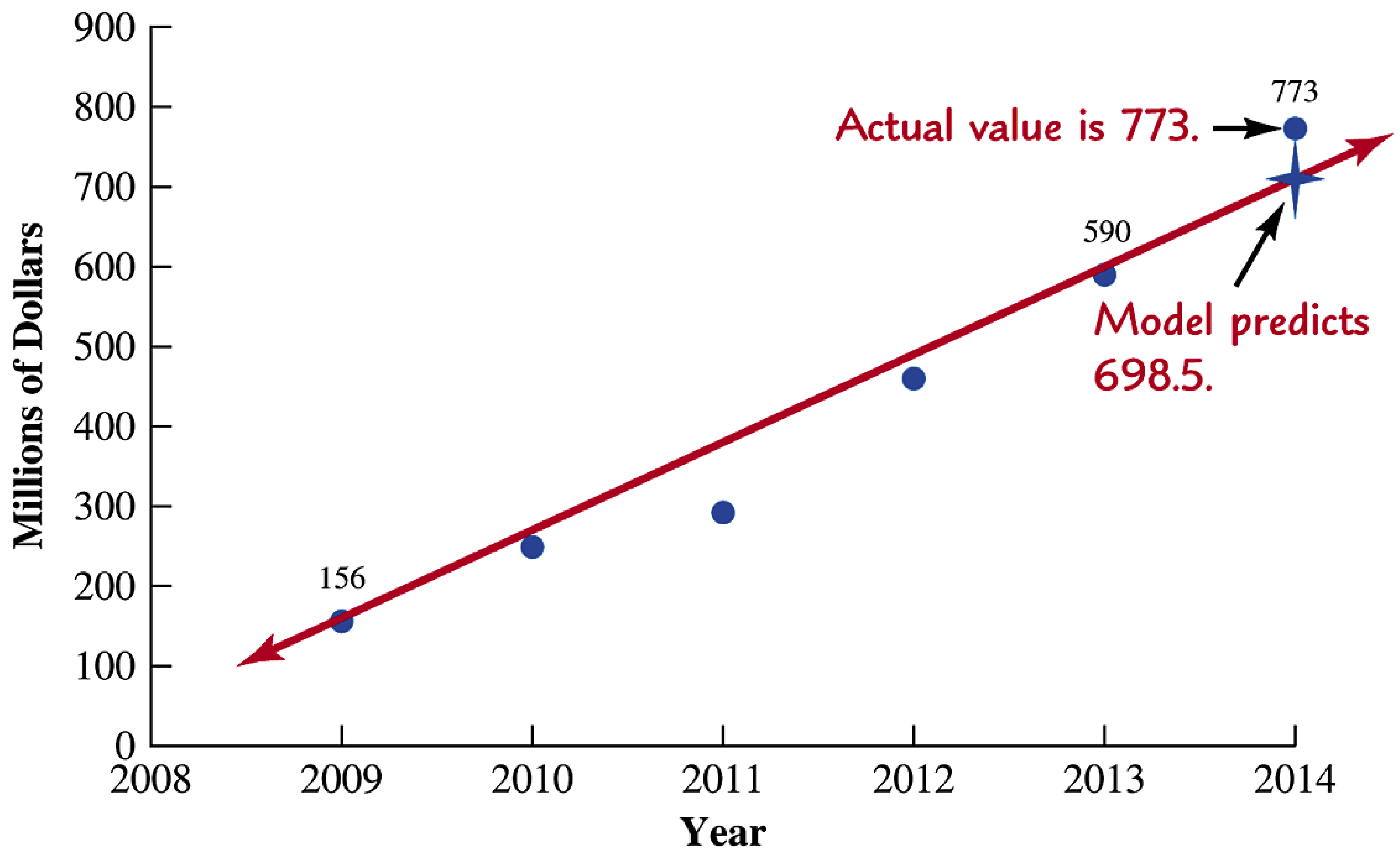
Example: Modeling Payments Made for Digital Radio Services (cont)



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