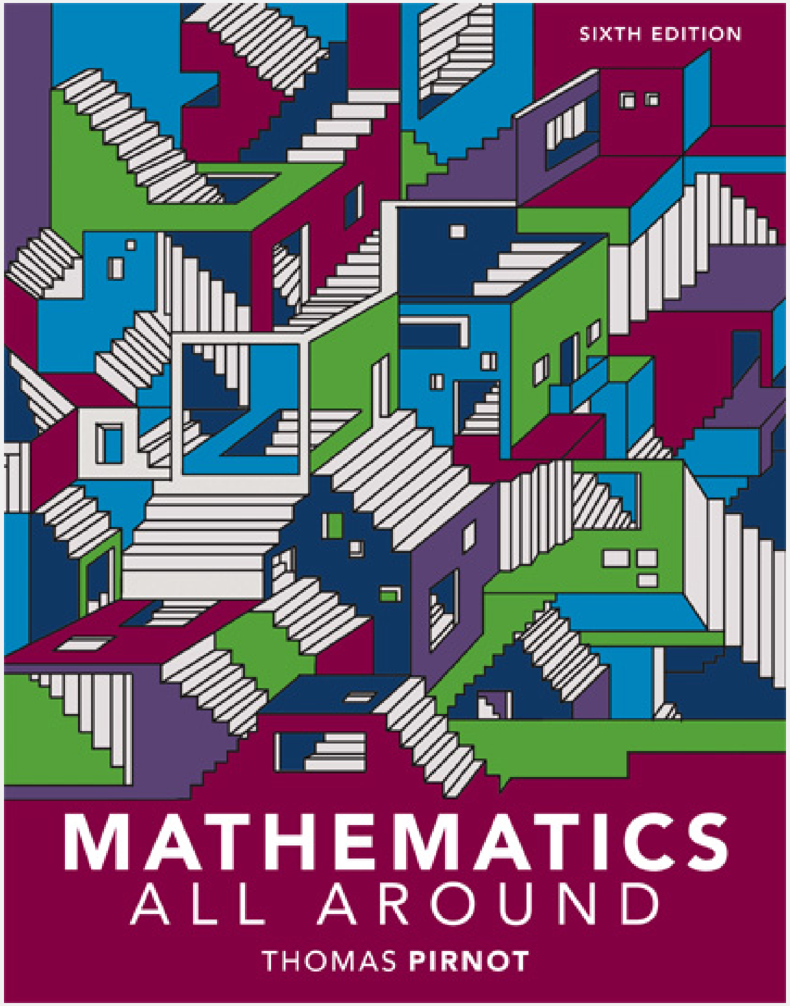
7.1

Algebraic Models: Linear Equations



7.1 Linear Equations

- Solve linear equations.
- Use intercepts to graph linear equations.
- Apply the slope-intercept form of a line in solving problems.

Solving Linear Equations

A **linear equation** in two variables is an equation that can be written in the form

$$Ax + By = C,$$

where *A, B,* and *C* are real numbers and *A* and *B* are not both zero. When a linear equation is written in this form, it is in **standard form.**

Solving Linear Equations

A *solution* for an equation is a number or numbers such that if we substitute them for the variables in the equation, the resulting statement is true.

The ordered pair (3, 15) is a solution for the equation $-3x + y = 6$ because

$$-3 \cdot 3 + 15 = 6$$

is *true*.

Solving Linear Equations

Two equations are *equivalent* if they have the same solutions.

In solving an equation, we use the following rules to rewrite it in a simpler, equivalent form.

1. You can add or subtract the same expression from both sides of an equation to get an equivalent equation.
2. You can multiply or divide both sides of an equation by the same *nonzero* expression to get an equivalent equation.

Example: Rewriting Equations in Equivalent Forms

Solve the equation $6x + 4y = 12$ for y.

Solution

$$6x + 4y = 12$$

$$4y = 12 - 6x$$

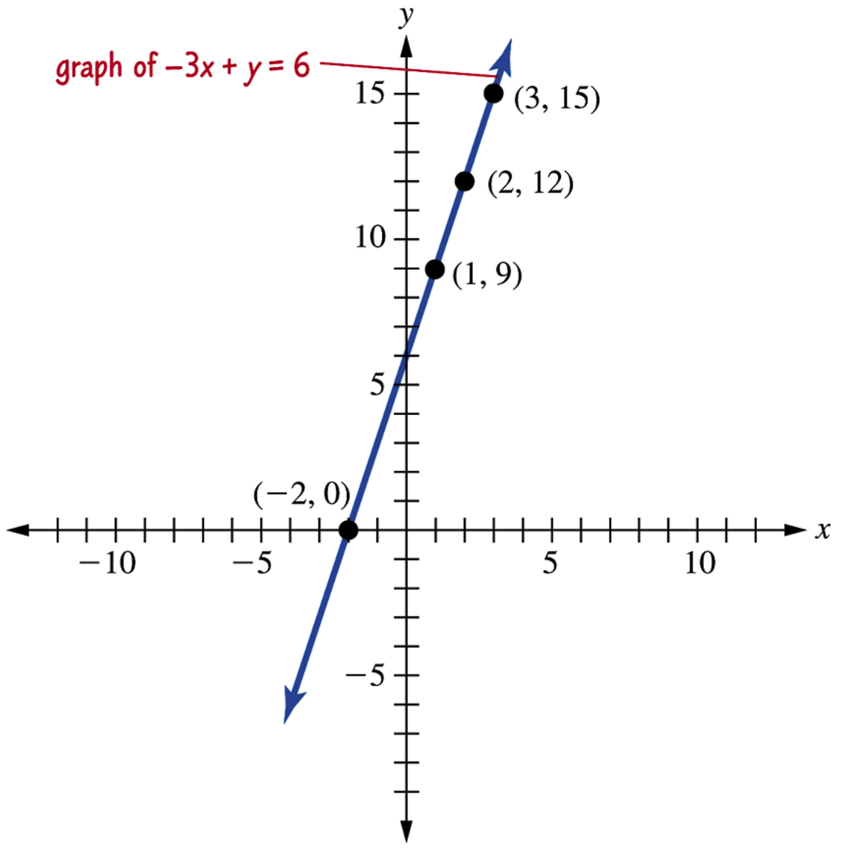
$$y = \frac{12 - 6x}{4}$$

$$y = \frac{12}{4} - \frac{6}{4}x$$

$$y = 3 - \frac{3}{2}x$$

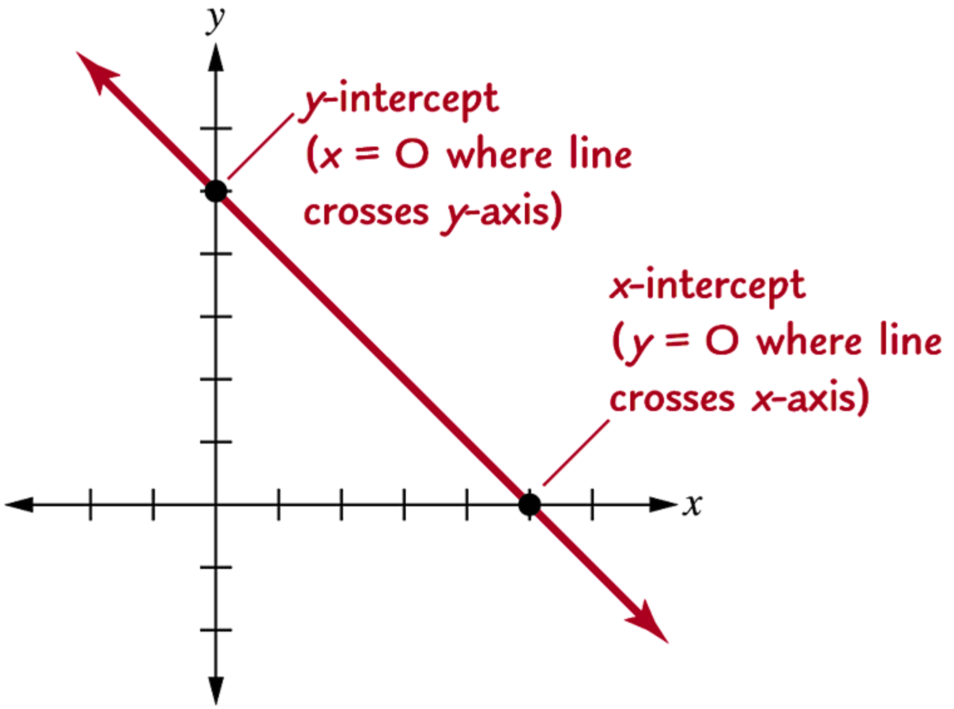
Graphing Linear Equations

The graph of a linear equation is always a straight line.



Intercepts

The ***x*-intercept** of the graph of a linear equation is the point where the graph crosses the *x*-axis. The ***y*-intercept** is the point where the graph crosses the *y*-axis. (See the figure.)



Example: Using Intercepts to Graph a Linear Equation

Find the intercepts and graph the equation $3x + 5y = 20$.

Solution

To find the x-intercept, we set $y = 0$.

$3x + 5(0) = 20$

$3x = 20$

$x = 6\ 2/3$

x-intercept: $(6\ 2/3, 0)$

Example: Using Intercepts to Graph a Linear Equation (cont)

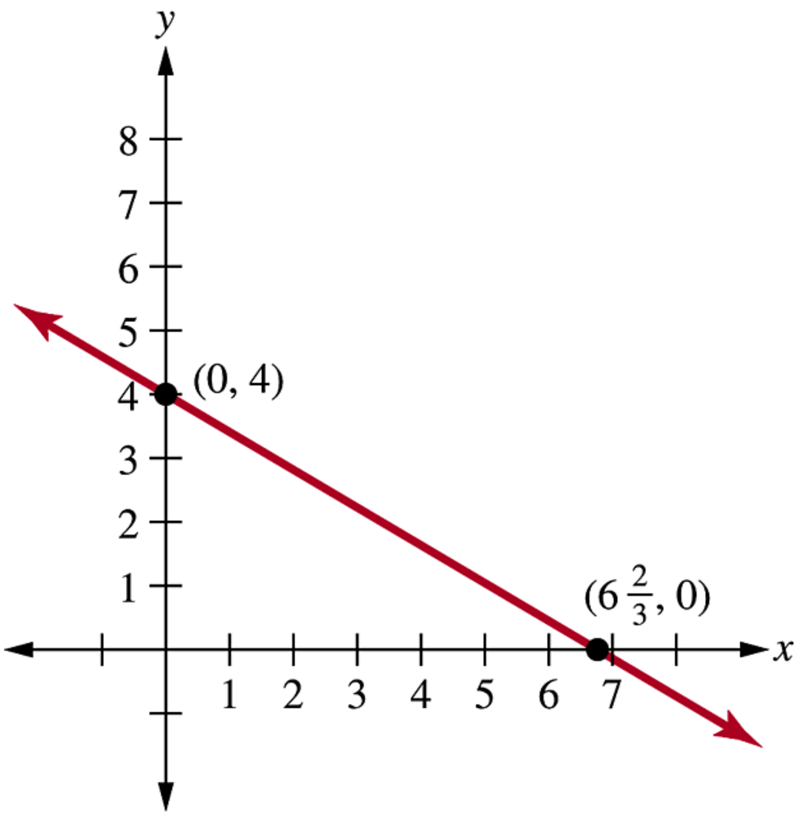
To find the *y*-intercept set $x = 0$.

$$3(0) + 5y = 20$$

$$5y = 20$$

$$y = 4$$

y-intercept: (0, 4)



Slope

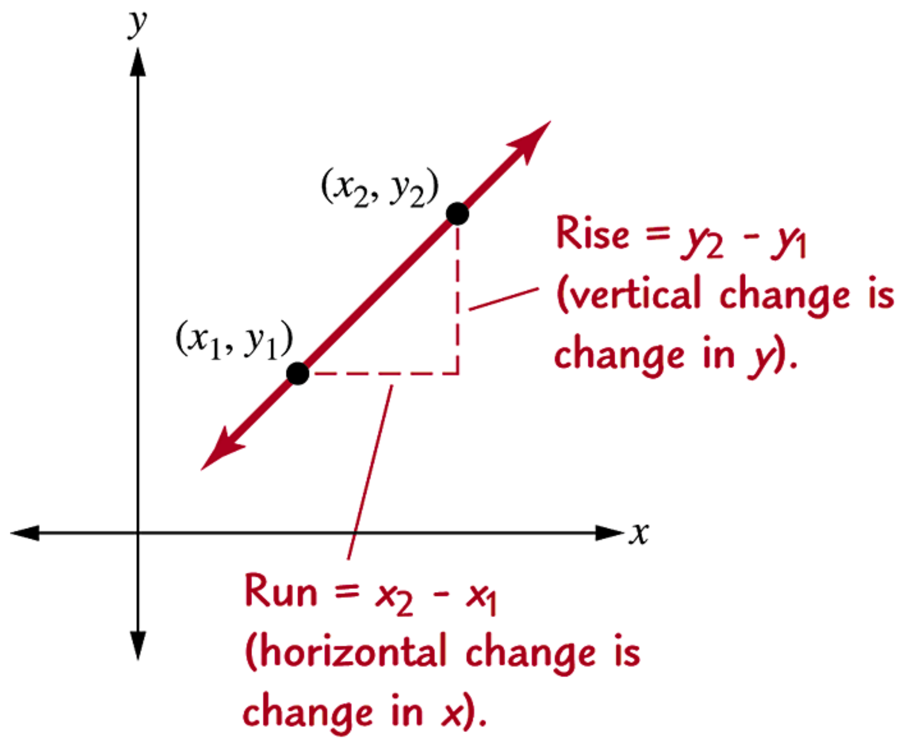
When using linear models, we often want to know the steepness, or *slope*, of an equation's graph.

Slope

If (x_1, y_1) and (x_2, y_2) are two points on a line and $x_1 \neq x_2$, then the **slope** of the line is defined as

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$



Example: Finding the Slope of a Line

Calculate the slope of the line that contains the points (1, 5) and (6, 45).

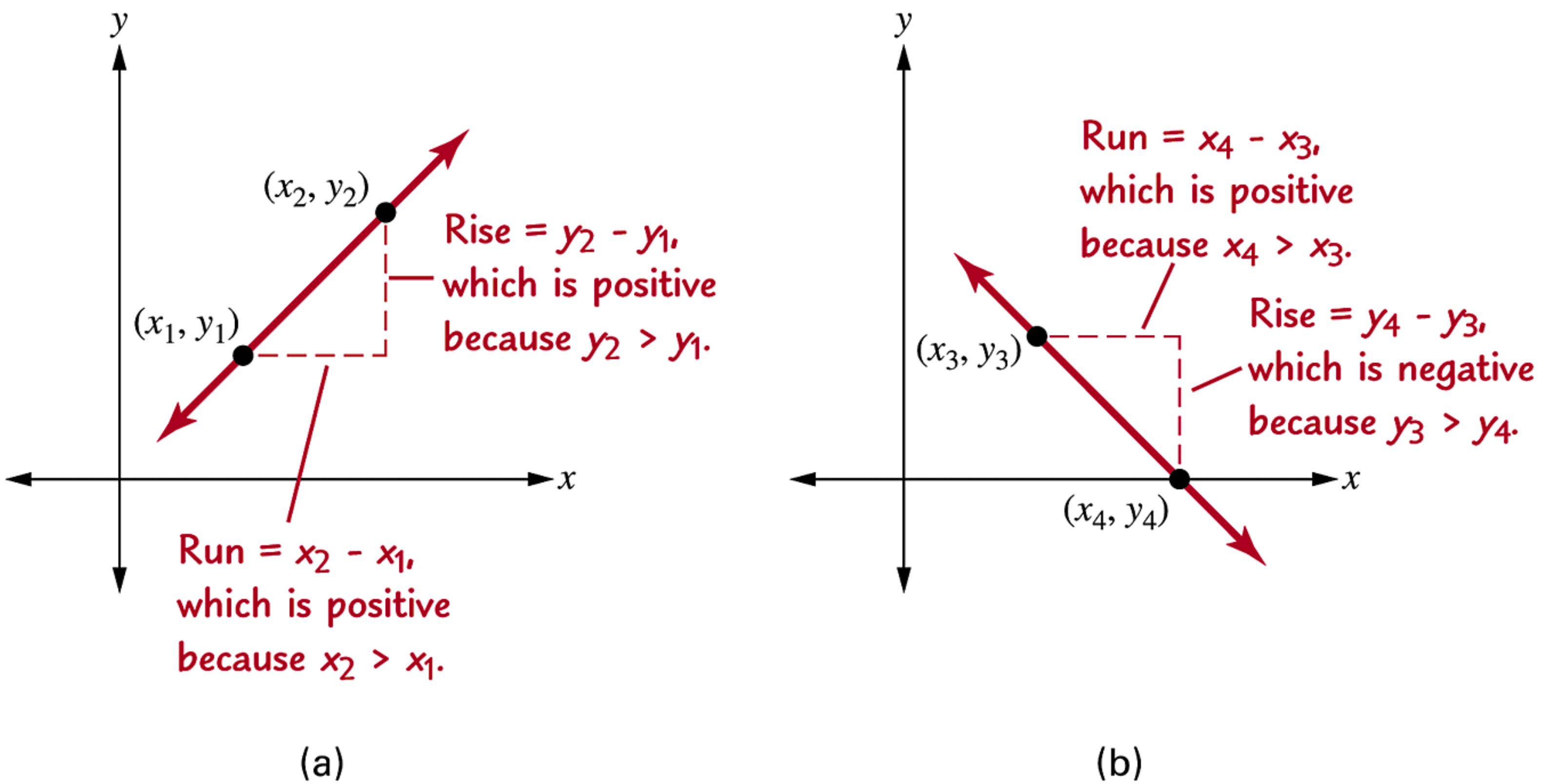
Solution

The slope of the line is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{45 - 5}{6 - 1} = \frac{40}{5} = 8.$$

Slope

Slope may be positive or negative.



Slope

Horizontal Line

The slope of a horizontal line is zero.

$$slope = \frac{rise}{run} = \frac{0}{run} = 0$$

Vertical Line

The slope of a vertical line is undefined.

$$slope = \frac{rise}{run} = \frac{rise}{0} \quad \text{undefined}$$

The Slope-Intercept Form of a Line

A linear equation is in **slope–intercept form** if it is written in the form $y = mx + b$. The number m is the slope of the line that is the graph of the equation, and $(0, b)$ is the y-intercept.

The equation $y = 2x + 3$ is in *slope-intercept form*.

The *slope* is 2 and the *y-intercept* is (0, 3).

Example: Graphing a Linear Equation in Slope–Intercept Form

Graph the linear equation $y = 3x + 4$.

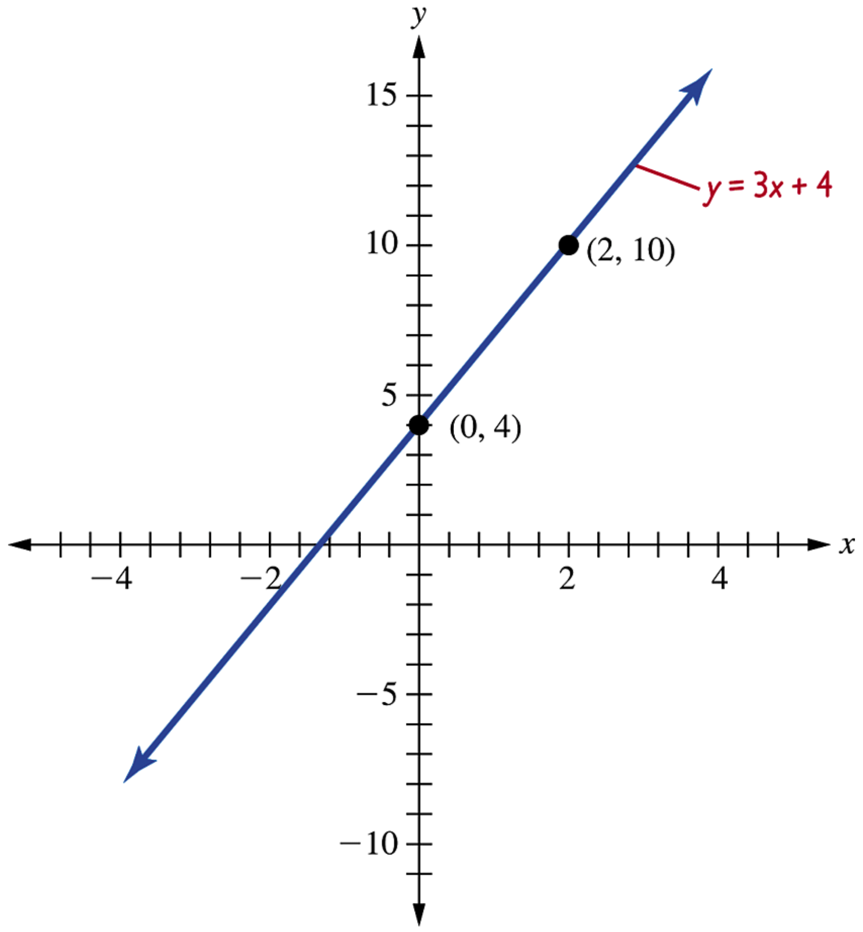
Solution

The y-intercept is (0, 4).

To find another point, let $x = 2$:

$y = (3)(2) + 4 = 10$

(2, 10)



Example: Comparing Stand Up Paddle Board Rental Plans

A company offers two different plans for renting SUP boards. With plan A, you pay a base fee of $35.00 plus $12.50 per hour. Plan B is a base fee of $22.50 plus $15 per hour. Model each plan using a linear equation in slope–intercept form. Graph the equations and estimate at what point plan A is more economical than plan B.

Example: Comparing Stand Up Paddle Board Rental Plans (cont)

Solution:

Plan A: $35.00 plus $12.50 per hr

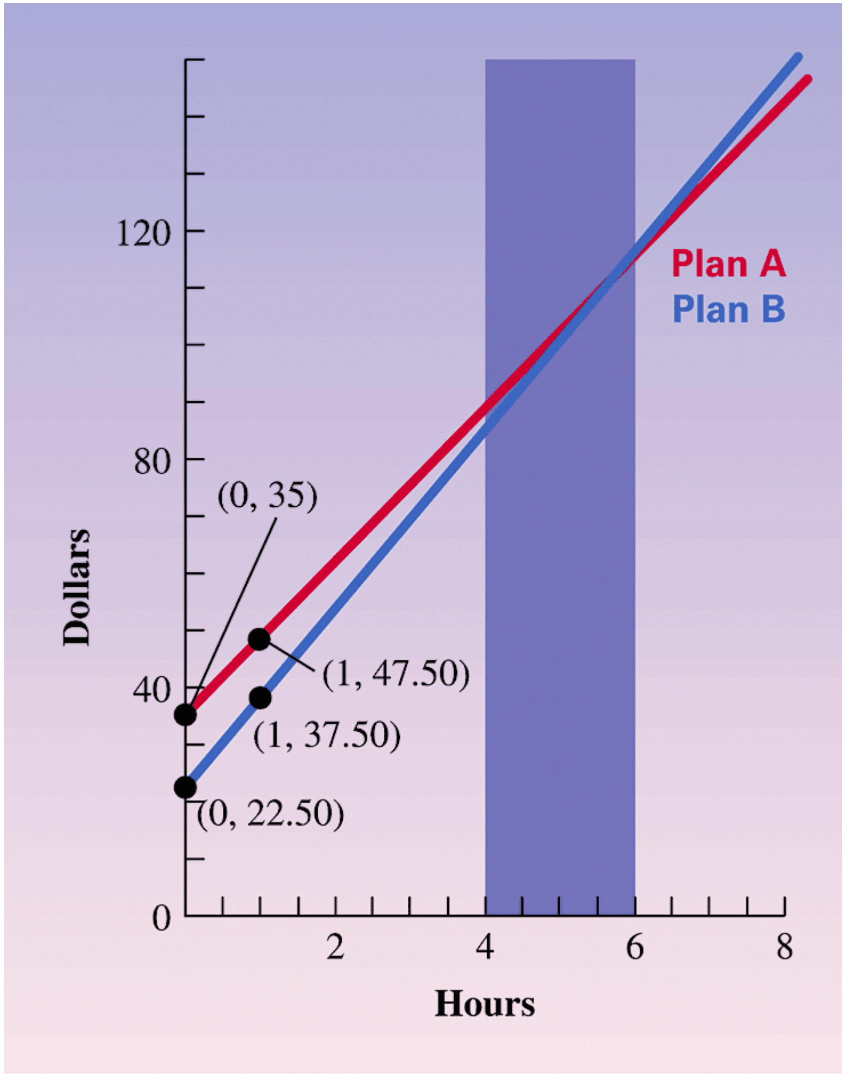
Plan B: $22.50 plus $15 per hr

The cost of renting a SUP board for x hours using plan A is the base fee of $35 plus x times $12.50. If we let y represent the total rental cost of a SUP board, we can model plan A by the equation

$$y = 12.50x + 35$$

Plan B is modeled by $y = 15x + 22.50$

Example: Comparing Stand Up Paddle Board Rental Plans (cont)



Example: Comparing Stand Up Paddle Board Rental Plans (cont)

We can see from the graph that for at least 4 hours, B’s graph lies below A’s, so plan B is cheaper, but by the time we reach 6 hours, B has become more expensive. So, it appears from the graphs that at around 5 hours, B becomes the more expensive plan.