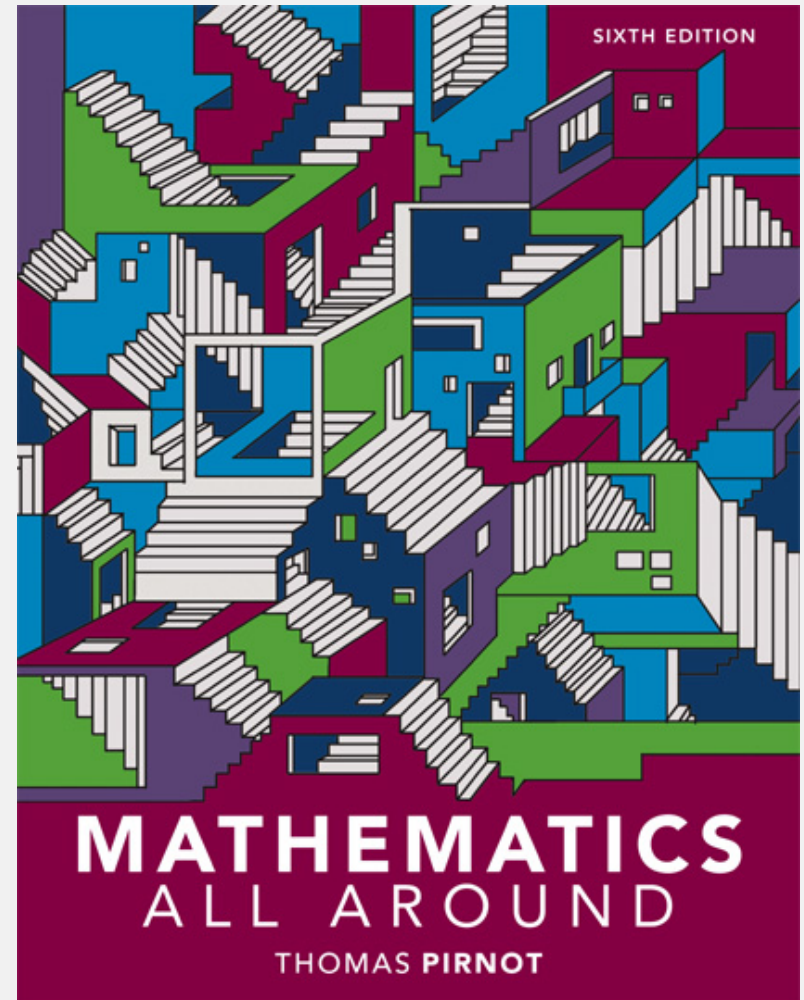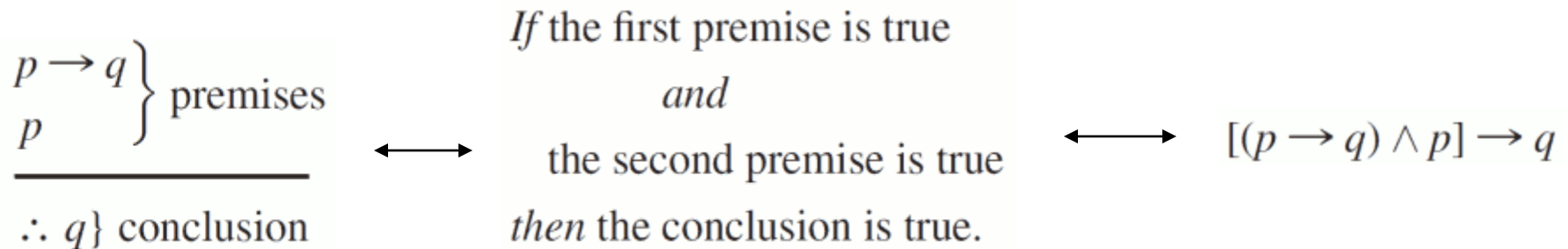# 3.4

Verifying Arguments

3.4   Verifying Arguments

- Use truth tables to show an argument to be valid.
- Use truth tables to verify arguments.
- Recognize some common valid argument forms.
- Recognize some common fallacies.

Verifying Arguments

An **argument** is a series of statements called **premises** followed by a single statement called the **conclusion.** An argument is **valid** if whenever all the premises are true, then the conclusion must also be true.

Representations of arguments:

$$p \rightarrow q \\ \underline{p} \\ \therefore q$$

$p \rightarrow q$, p } premises

$\therefore q$ } conclusion

$\longleftrightarrow$

If the first premise is true

and

the second premise is true

then the conclusion is true.

$\longleftrightarrow$

$[(p \rightarrow q) \wedge p] \rightarrow q$

Verifying an Argument

1. Make a truth table with separate columns for each premise and the conclusion.

2. Examine *only* the lines in the table in which all of the premises are true.

3. If the conclusion is also true for the lines you examined in step 2, the argument is valid.

4. If the conclusion is false for even one of the lines you examined in step 2, then the argument is invalid.

Example: Determining the Validity of an Argument

Determine whether the following argument is valid:

If you buy the more expensive Hulu plan, then you can watch TV commercial free.

You cannot watch TV commercial free.

∴ You did not buy the more expensive Hulu plan.

Example: Determining the Validity of an Argument (cont)

Solution

Let *e* stand for "You buy the more expensive Hulu plan" and let *f* represent "You can watch TV commercial free." Then this argument has the following form:

$$e \rightarrow f$$

$$\underline{\sim f}$$

$$\therefore \sim e$$

Example: Determining the Validity of an Argument (cont)

Now we make a truth table for this argument.

e	f	$e \rightarrow f$	$\sim f$	$\sim e$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Because the only line in which both premises are true also has a true conclusion, we conclude that this argument is valid.

Example: Some Invalid Arguments

Determine whether the following argument is valid:

If I have the new calling plan, then I can text my friends for free.

I can text my friends for free.

Therefore, I have the new calling plan.

Example: Some Invalid Arguments (cont)

Solution

If we let *c* represent "I have a new calling plan" and let *f* represent "I can text my friends for free," then we can write the argument symbolically as

$$c \rightarrow f$$
$$\underline{f}$$
$$\therefore c$$

Example: Some Invalid Arguments (cont)

We will make a truth table, as we did in a previous example, with columns for both premises and the conclusion.

c	f	$c{\rightarrow}f$	f	c
T	T	T	T	
T	F	F	F	
F	T	T	T	F
F	F	T	F	

Line 3 in the table shows that it is possible for both premises to be true, but the conclusion is false. This means that the argument is invalid.

Valid Argument Forms and Fallacies

Valid Arguments

Law of Detachment	Law of Contraposition	Law of Syllogism	Disjunctive Syllogism
$p \rightarrow q$ p $\overline{}$ $\therefore q$	$p \rightarrow q$ $\sim q$ $\overline{}$ $\therefore \sim p$	$p \rightarrow q$ $q \rightarrow r$ $\overline{}$ $\therefore p \rightarrow r$	$p \vee q$ $\sim p$ $\overline{}$ $\therefore q$

Invalid Arguments

Fallacy of the Converse	Fallacy of the Inverse
$p \rightarrow q$ q $\overline{}$ $\therefore p$	$p \rightarrow q$ $\sim p$ $\overline{}$ $\therefore \sim q$

Example: Identifying the Form of an Argument

Identify the form of each argument and state whether it is valid or invalid.

If you want to improve your cardiovascular fitness, then take up cross-country skiing.

You take up cross-country skiing.

Therefore, you want to improve your cardiovascular fitness.

Example: Identifying the Form of an Argument (cont)

Solution

Let *i* represent "You want to improve your cardiovascular fitness" and let *c* represent "You take up cross-country skiing." This argument has the following form:

$$i \rightarrow c$$

$$\frac{c}{\therefore i}$$

This is the fallacy of the converse, which is an invalid argument form.

Example: Identifying the Form of an Argument

Identify the form of each argument and state whether it is valid or invalid.

If you sleep through your morning math class, then you will be well rested.

If you are well rested, then you will do well on your math test.

Therefore, if you sleep through your morning math class, then you will do well on your math test.

Example: Identifying the Form of an Argument (cont)

Solution

Let *s* represent "You sleep through your morning math class," let *r* represent "You will be well rested," and let *w* represent "You will do well on your math test." The argument then has the following form:

$$s \rightarrow r$$

$$\underline{r \rightarrow w}$$

$$\therefore s \rightarrow w$$

This is a syllogism and is therefore valid, even though the reasoning seems to make no sense.

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