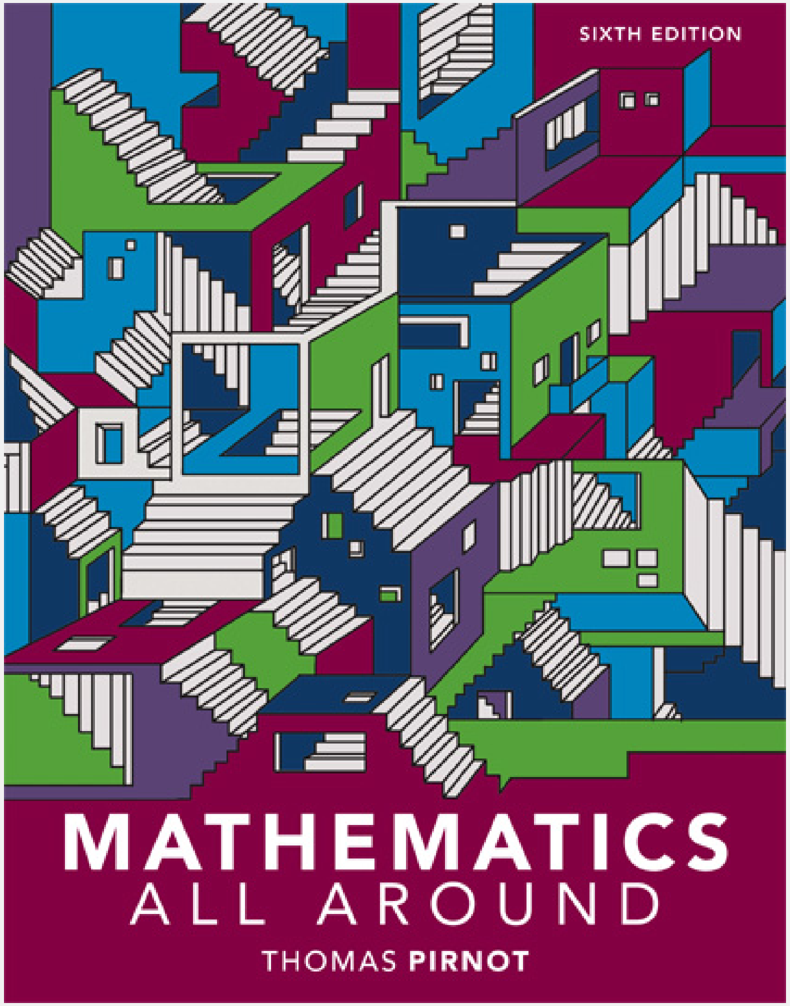
3.2

Logic



3.2 Truth Tables

- State the truth tables for the five fundamental connectives.
- Compute truth tables for compound statements.
- Determine when statements are logically equivalent.
- State and apply DeMorgan's laws.

Truth Tables

We want to know if a pair of similar statements such as $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ mean the same thing.

We use *truth tables* to determine when compound statements are true and when they are false, and whether a pair of statements have the same meaning.

Truth Tables - Negation

If *p* is true, then ~*p* is false.

If *p* is false, then ~*p* is true.

p	$\sim p$
T	F
F	T

all possible truth values for statement *p*

logical truth values for ~*p*

Example: Finding the Truth Value of Negations

Determine whether each statement is true or false and then state its negation. Notice how the negation of each statement has the opposite truth value of the original statement.

a) Maya Angelou is a well-known American poet.

b) The Bill of Rights begins the Constitution of the United States.

Example: Finding the Truth Value of Negations (cont)

a) Maya Angelou is a well-known American poet.

This is true. Therefore, its negation, “Maya Angelou is not a well-known American poet.,” is false.

Example: Finding the Truth Value of Negations (cont)

b) The Bill of Rights begins the Constitution of the United States.

This is false. So, “The Bill of Rights does not begin the Constitution of the United States.” is true.

Truth Tables - Conjunction

A conjunction is true only when both of its components are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

all possible ways that p and q can be true or false

Truth Tables - Disjunction

A disjunction is false only when both *p* and *q* are false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

all possible ways that *p* and *q* can be true or false

inclusive or

Example: Analyzing the Logic of an Advertisement

Consider the following help-wanted ad:

"Management trainee wanted. Applicant must have four-year degree in accounting or three years of experience working in a financial institution."

Which applicants should be considered for the position if we interpret *or* according to the table?

Example: Analyzing the Logic of an Advertisement (cont)

a) Aya has a four-year degree in accounting and has worked two years for a loan company.

b) Heidi has studied accounting in college (but did not graduate) and has worked for five years selling electronics.

c) Monte earned a four-year degree in accounting and has five years of experience working for a credit card company.

Example: Analyzing the Logic of an Advertisement (cont)

Solution

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

all possible ways that p and q can be true or false

inclusive *or*

a) Aya's background corresponds to line 2 of the table, so she should be considered for the job.

b) Heidi's credentials are reflected in line 4 of the table, so she is not qualified.

c) Monte's background corresponds to line 1 of the table, so he should be considered.

Truth Tables

Because of their similar structures, there are parallels that occur in logic and set theory.

Logic		Set Theory	
and	$\wedge$	$\cap$	intersection
or	$\vee$	$\cup$	union
not	$\sim$	$'$	complement
if … then	$\rightarrow$	$\subseteq$	subset

Often if a result is true for set theory, then a similar result also holds for logic (and vice versa).

Compound Statements

We use truth tables to find the logical values of complex statements.

Example: Finding the Truth Table for a Compound Statement

Compute a truth table for a statement of the form $(\sim p \wedge q) \vee (p \wedge q)$.

Solution

So we can solve this problem easily by breaking it into three simpler problems:

Step 1 Calculate $(\sim p \wedge q)$.

Step 2 Calculate $(p \wedge q)$.

Step 3 Use the results from steps 1 and 2 to calculate $(\sim p \wedge q) \vee (p \wedge q)$.

Example: Finding the Truth Table for a Compound Statement (cont)

We will next perform each of these steps in the following truth table:

p	q	$\sim p$	$\sim p \wedge q$ (Step 1)	$p \wedge q$ (Step 2)	$(\sim p \wedge q) \vee (p \wedge q)$ (Step 3)
T	T	F	F	T	**T**
T	F	F	F	F	**F**
F	T	T	T	F	**T**
F	F	T	F	F	**F**

If *p* is false and *q* is true, line 3 tells us that $(\sim p \wedge q) \vee (p \wedge q)$ is true.

Compound Statements

Definition: If the final column of a truth table contains all T’s, then the statement is always true. Such a statement is called a *tautology*.

Compound Statements

How many lines should your truth table have?

If the statement has k variables, then its truth table will have 2^k lines.

Logically Equivalent Statements

Two statements are **logically equivalent** if they have the same variables and, when their truth tables are computed, the final columns in the tables are identical.

Example: Determining When Statements Mean the Same Thing

Assume that you have an entertainment book containing discount coupons for movies, restaurants, and other leisure activities. You are considering eating at either the Pasta Bar or the Deli. Do the following two statements say the same thing?

a) Both the Pasta Bar and the Deli accepts coupons is false.

b) The Pasta Bar does not accept coupons or the Deli does not accept coupons.

Example: Determining When Statements Mean the Same Thing (cont)

Solution

If we let *p* represent “The Pasta Bar accepts coupons” and let *d* represent “The Deli accepts coupons,” then we can write a) and b) symbolically as

$$a) \sim (p \wedge d)$$

$$b) (\sim p) \vee (\sim d)$$

To decide whether these statements say the same thing, all we have to do is make two truth tables.

Example: Determining When Statements Mean the Same Thing (cont)

p	d	$p \wedge d$	$\sim(p \wedge d)$
T	T	T	**F**
T	F	F	**T**
F	T	F	**T**
F	F	F	**T**

$\sim p$	$\sim d$	$(\sim p) \vee (\sim d)$
F	F	**F**
F	T	**T**
T	F	**T**
T	T	**T**

Because the final columns (shaded) in the two truth tables are identical, the two statements are logically equivalent and therefore express exactly the same information.

DeMorgan's Laws for Logic

If *p* and *q* are statements, then

a) $\sim(p \wedge q)$ is logically equivalent to $(\sim p) \vee (\sim q)$.

b) $\sim(p \vee q)$ is logically equivalent to $(\sim p) \wedge (\sim q)$.

Example: Applying Logic to Legal Documents

Use DeMorgan's laws to rewrite the following statement, which is based on instructions for filing Form 1040-A with the U.S. Internal Revenue Service.

You received interest from a seller-financed mortgage and the buyer used the property as a personal residence is false.

Example: Applying Logic to Legal Documents (cont)

Solution

It is easy to rewrite this statement if first we represent it in symbolic form. Let *r* represent "You received interest from a seller-financed mortgage" and let *b* represent "The buyer used the property as a personal residence." This statement has the form $\sim(r \wedge b)$.

By DeMorgan's laws this statement is equivalent to $(\sim r) \vee (\sim b)$. We can now rewrite this in English as "You did not receive interest from a seller-financed mortgage or the buyer did not use the property as a personal residence."

Example: An Alternative Method to Construct Truth Tables

a) We will construct a truth table for $(\sim p \wedge q) \vee (p \wedge q)$, which you saw in a previous example.

Recall that we constructed a table by performing the following steps.

Step 1 Calculate $(\sim p \wedge q)$.

Step 2 Calculate $(p \wedge q)$.

Step 3 Use the results from steps 1 and 2 to calculate $(\sim p \wedge q) \vee (p \wedge q)$.

Example: An Alternative Method to Construct Truth Tables (cont)

			Step 1		Step 3		Step 2	
p	q	$(\sim p$	$\wedge$	$q)$	$\vee$	$(p$	$\wedge$	$q)$
T	T	F	F	T	**T**	T	T	T
T	F	F	F	F	**F**	T	F	F
F	T	T	T	T	**T**	F	F	T
F	F	T	F	F	**F**	F	F	F

Copy values for p and q.

Example: An Alternative Method to Construct Truth Tables (cont)

b) We will construct a truth table for $(\sim p \vee q) \wedge (\sim r)$ by performing the calculations as follows:

Step 1: Calculate $(\sim p \vee q)$.

Step 2: Calculate $\sim r$.

Step 3: Calculate $(\sim p \vee q) \wedge (\sim r)$.

Example: An Alternative Method to Construct Truth Tables (cont)

				Step 1		Step 3	Step 2
p	q	r	$(\sim p$	$\vee$	$q)$	$\wedge$	$(\sim r)$
T	T	T	F	T	T	**F**	F
T	T	F	F	T	T	**T**	T
T	F	T	F	F	F	**F**	F
T	F	F	F	F	F	**F**	T
F	T	T	T	T	T	**F**	F
F	T	F	T	T	T	**T**	T
F	F	T	T	T	F	**F**	F
F	F	F	T	T	F	**T**	T

Use these values to compute step 3.

Assignment

Do 3.2.38 and turn in next class. Show how you came to your conclusion by building a Truth Table.