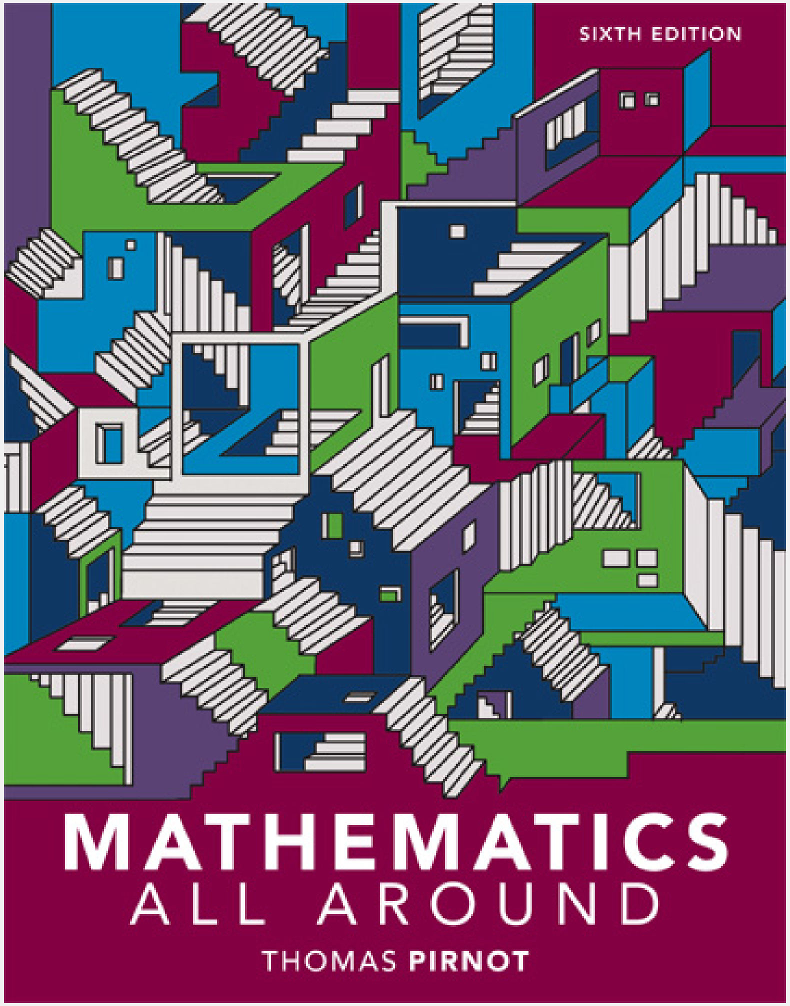
3.1

Logic



3.1 Statements, Connectives, and Quantifiers

- Identify statements in logic.
- Represent statements symbolically using five connectives.
- Recognize the difference between the universal and existential quantifiers.
- Write the negations of quantified statements.

Statements in Logic

In symbolic logic, we only care whether statements are true or false.

A **statement** in logic is a declarative sentence or assertion that is either true or false (but not both). We represent statements by lowercase letters such as *p*, *q*, or *r*.

Statements in Logic

Examples of statements- Remember that to be a statement, the sentence must be a declaration or an assertion that is either true or false.

a) Excessive cellphone use among young people is causing psychoses.

b) If you eat less and exercise more, you will lose weight.

c) In the last 10 years, we have reduced by 25% the amount of greenhouse gases in the atmosphere.

Statements in Logic

The following are not statements:

e) Come here. (This is not a declarative sentence.)

f) When did dinosaurs become extinct? (This is not a declarative sentence.)

g) This statement is false. (This is a paradox. It cannot be either true or false. If we assume that this sentence is true, then we must conclude that it is false. On the other hand, if we assume it is false, then we conclude that it must be true.)

Statements in Logic

A **simple statement** contains a single idea. A **compound statement** contains several ideas combined together. The words used to join the ideas of a compound sentence are called **connectives.**

Connectives

The connectives used in logic generally fall into five categories:

- *Negation*
- *Conjunction*
- *Disjunction*
- *Conditional*
- *Biconditional*

Connectives

Negation is a statement expressing the idea that something is not true. We represent negation by the symbol ~ (a Tilde).

Example: Negating Sentences

Negate each statement:

a) *r*: The retina display is included in the price of your MacBook.

The retina display is *not* included in the price of your MacBook. We write this negation symbolically as ~r.

b) *b*: The blue whale is the largest living creature.

Example: Negating Sentences (cont)

b) *b*: The blue whale is the largest living creature.

The blue whale is *not* the largest living creature. The symbols ~*b* represent this statement.

Connectives

A **conjunction** expresses the idea of *and*. We use the symbol $\wedge$ to represent a conjunction.

Example: Joining Statements Using "And"

Consider the following statements:

p: The tenant pays utilities.

d: A $150 deposit is required.

a) Express the statement "It is not true that: the tenant pays utilities and a $150 deposit is required" symbolically.

b) Write the statement $\sim p \wedge \sim d$ in English.

Example: Joining Statements Using "And" (cont)

Solution

a) "It is not true that: the tenant pays utilities and a $150 deposit is required" symbolically

This sentence has the form $\sim(p \wedge d)$.

b) The tenant does not pay utilities and a $150 deposit is not required.

Problem Solving

Strategy: The Order Principle

Notice the difference in the form of the statements in Example 2 that we show in this diagram:

First use "and."

a) $\sim(p \wedge d)$

Then negate

First negate.

b) $\sim p \wedge \sim d$

Then use "and."

These statements sound similar, but do not say the same thing. Keep in mind that changing the order of logical operations can change the meaning of a statement.

Connectives

A **disjunction** conveys the notion of *or*. We use the symbol $\vee$ to represent a disjunction.

Example: Joining Ideas Using "Or"

Consider the following statements:

h: We will build more hybrid cars.

f : We will use more foreign oil.

Write the following statement symbolically:

We will not build more hybrid cars or we will use more foreign oil.

Solution

$$(\sim h) \vee f$$

Connectives

A **conditional** expresses the notion of *if . . . then*. We use an arrow, $\rightarrow$, to represent a conditional.

Example: Connecting Ideas Using "If . . . Then"

Suppose that p represents "The Phillies win the World Series" and m represents "Mayim will win an Emmy."

a) We would read the statement $p \rightarrow m$ as "If the Phillies win the World Series, then Mayim will win an Emmy."

b) We would write the statement "If the Phillies do not win the World Series, then Mayim will not win an Emmy" symbolically as $\sim p \rightarrow \sim m$.

Connectives

A **biconditional** represents the idea of *if and only if*. Its symbol is a double arrow, $\leftrightarrow$.

Example: Connecting Ideas with "If and Only If"

Write each biconditional statement in symbolic form:

a) A polygon has three sides if and only if it is a triangle.

Let p represent the statement "A polygon has three sides" and let *t* represent the statement "The polygon is a triangle." Then this statement has the form $p \leftrightarrow t$.

Example: Connecting Ideas with "If and Only If" (cont)

Write each biconditional statement in symbolic form:

b) The Blu-ray disc can be returned if and only if the seal is not broken.

Let r represent the statement "The Blu-ray disc can be returned" and let b represent the statement "The seal is broken." Then this statement has the form $r \leftrightarrow \sim b$.

Quantifiers

Quantifiers tell us "how many" and fall into two categories:

Universal Quantifiers and Existential Quantifiers.

Quantifiers

Universal quantifiers are words such as *all* and *every* that state that all objects of a certain type satisfy a given property.

Examples:

All citizens over age 18 have the right to vote.

Every triangle has an interior angle sum of 180 degrees.

Each NASCAR driver must register for the Daytona 500 by August 1.

Quantifiers

Existential quantifiers are words such as *some*, *there exists*, and *there is at least one* that state that there are one or more objects that satisfy a given property.

Examples:

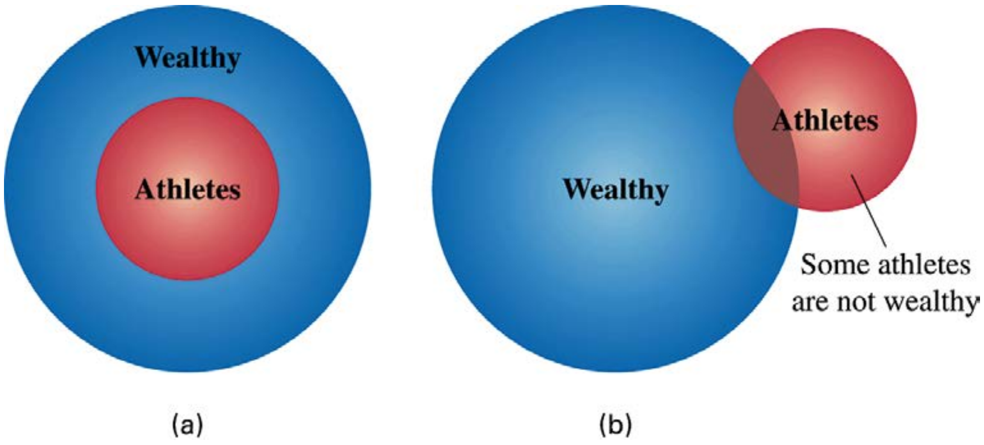
Some drivers qualify for reduced insurance rates.

There is a number whose square is 25.

There exists a bird that cannot fly.

Negating Quantifiers

Suppose we want to negate the statement "All professional athletes are wealthy." (universal)

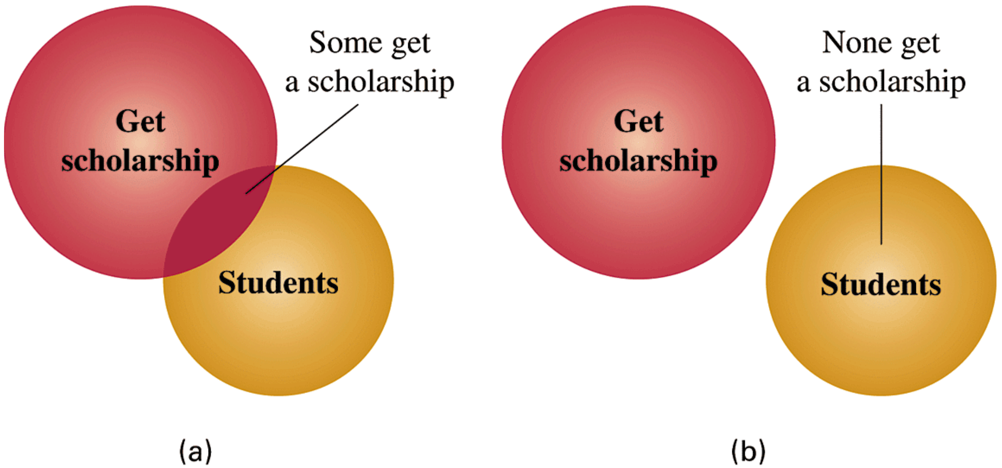


Correct: "Some athletes are not wealthy" or "Not all athletes are wealthy." (both existential)

Incorrect: "All athletes are not wealthy." (universal)

Negating Quantifiers

Negate the statement “Some students will get a scholarship.” (existential)



Correct: “No students will get a scholarship.” (universal)

Incorrect: “Some students will not get a scholarship.” (existential)

Negating Statements With Quantifiers

The phrase *Not all are* has the same meaning as *At least one is not*.

The phrase *Not some are* has the same meaning as *None are*.

Example: Negating Quantified Statements

Negate each quantified statement and rewrite it in English.

a) All customers will get a free dessert.

b) Some tablet computers have a two-year warranty.

Example: Negating Quantified Statements (cont)

Solution

a) All customers will get a free dessert.

We want to rewrite the statement "Not all customers will get a free dessert." Because this statement has the form *Not all are,* we can rewrite it as *At least one is not.* In English we would say "At least one customer will not get a free dessert."

Example: Negating Quantified Statements (cont)

Solution

b) Some tablet computers have a two-year warranty.

The negation of the statement is “It is not true that some tablet computers have a two-year warranty.” Because this statement has the form *Not some have,* we can rewrite it as *None have.* In English we would say “No tablet computers have a two-year warranty.”