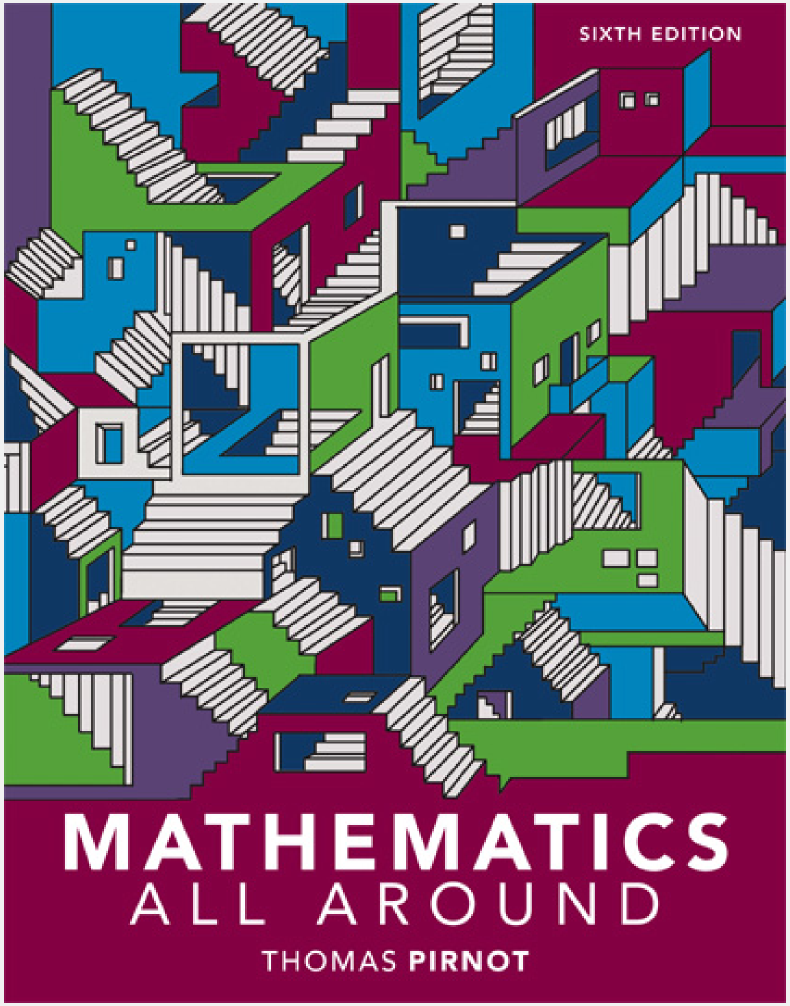
2.3

Set Theory



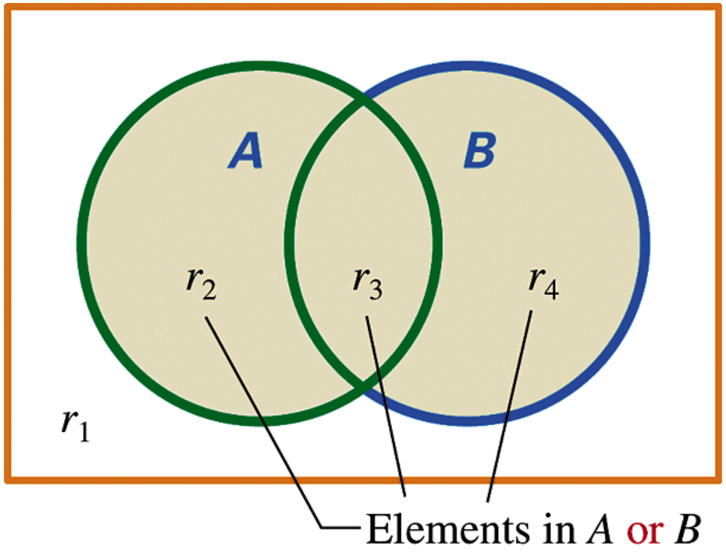
2.3 Set Operations

- Perform the set operations of union, intersection, complement, and difference.
- Be able to perform set operations in the proper order.
- Be able to apply DeMorgan’s laws in set theory.
- Use the Inclusion-Exclusion Principle to calculate the cardinal number of the union of two sets.
- Use Venn diagrams to prove or disprove set theory statements.

Union of Sets

The **union** of sets *A* and *B*, written $A \cup B$, is the set of elements that are members of either *A* or *B* (or both). In set-builder notation, $A \cup B = \{x : x$ is a member of A or x is a member of $B\}$.

The union of more than two sets is the set of all elements belonging to at least one of the sets.



Example: Finding the Union of Sets

Find the union of the following pairs of sets.

$A = \{1, 3, 5, 6, 8\}$, $B = \{2, 3, 6, 7, 9\}$

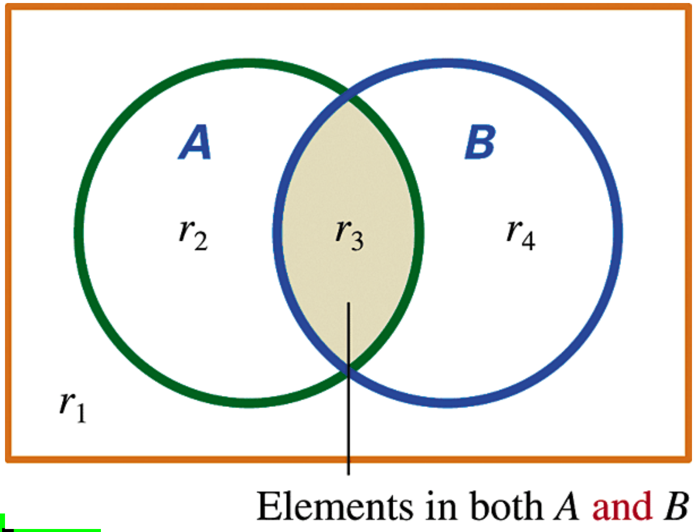
Solution

$A \cup B = \{1, 3, 5, 6, 8\} \cup \{2, 3, 6, 7, 9\}$ = the set of elements that are in *A* or *B* or both = $\{1, 3, 5, 6, 8, 2, 3, 6, 7, 9\} = \{1, 2, 3, 5, 6, 7, 8, 9\}$. Notice in our final answer how we listed the elements *in order* and *did not list duplicate elements* because doing so does not affect set equality.

Intersection of Sets

The **intersection** of sets *A* and *B*, written $A \cap B$, is the set of elements that are common to both *A* and *B*. In set-builder notation,
$A \cap B = \{x : x \text{ is a member of } A \text{ and } x \text{ is a member of } B\}$.

The intersection of more than two sets is the set of elements that belong to each of the sets. If $A \cap B = \varnothing$, then we say that *A* and *B* are **disjoint**.



Example: Finding the Intersection of Sets

Find the intersection of the following pairs of sets.

$A = \{1, 3, 5, 6, 8\}$, $B = \{2, 3, 6, 7, 9\}$

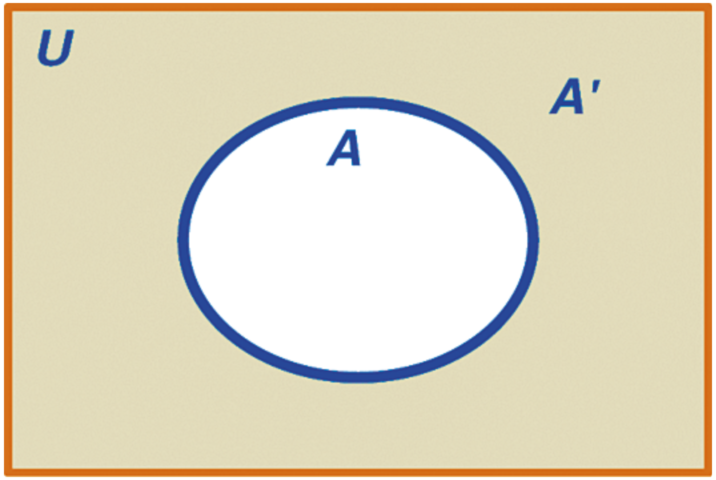
Solution

$A \cap B = \{1, 3, 5, 6, 8\} \cap \{2, 3, 6, 7, 9\}$ = the set of elements that are in A and B = $\{3, 6\}$

Set Complement

If A is a subset of the universal set U, the **complement** of A is the set of elements of U that are *not* elements of A. This set is denoted by A' *(pronounced A prime)*. In set-builder notation,

$$A' = \{x : x \in U \text{ but } x \notin A\}.$$



Example: Finding the Complement of Sets

Find the complement of each set. We have stated a universal set for each set.

a) $U = \{1, 2, 3, \ldots, 10\}$ and $A = \{1, 3, 5, 7, 9\}$

b) *U* is the set of people living in the United States, and *T* is the set of people who have photos on Instagram.

c) *U* is the set of cards in a standard 52-card deck, and *F* is the set of face cards.

Example: Finding the Complement of Sets (cont)

Solution

a) $U = \{1, 2, 3, \ldots, 10\}$ and $A = \{1, 3, 5, 7, 9\}$

A' is the set of elements in U that are not in A, so $A' = \{2, 4, 6, 8, 10\}$.

b) U is the set of people living in the United States, and T is the set of people who have photos on Instagram.

T' = is the set of people living in the United States who do not have photos on Instagram.

Example: Finding the Complement of Sets (cont)

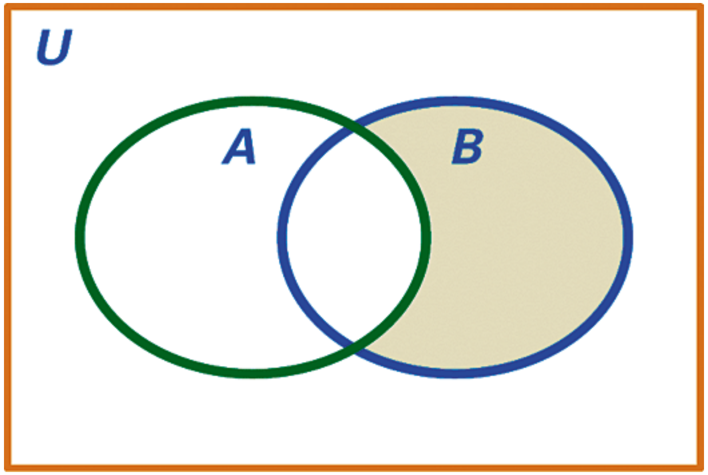
c) *U* is the set of cards in a standard 52-card deck, and *F* is the set of face cards.

F' = is the set of nonface cards.

Set Difference

The **difference** of sets B and A is the set of elements that are in B but not in A. This set is denoted by $B - A$. In set-builder notation,

$B - A = \{x : x$ is a member of B and x is not a member of $A\}$.



Example: Finding the Difference of Sets

Find {3, 6, 9, 12} – {*x*: *x* is an odd integer}.

Solution

We start with {3, 6, 9, 12} and remove all the odd integers to get {6, 12}.

Order of Set Operations

Just as we must perform arithmetic operations in a certain order, set notation specifies the order in which we perform set operations.

Example: Order of Set Operations

Let $U = \{1, 2, 3, \ldots, 10\}, E = \{x : x \text{ is even}\},$
$B = \{1, 3, 4, 5, 8\},$ and $A = \{1, 2, 4, 7, 8\}.$

Find $(A \cup B)' \cap (E' \cup A).$

Solution

1. $(A \cup B) = \{1, 2, 3, 4, 5, 7, 8\}$
2. $(A \cup B)' = \{6, 9, 10\}$
3. $E' = \{1, 3, 5, 7, 9\}$
4. $(E' \cup A) = \{1, 2, 3, 4, 5, 7, 8, 9\}$
5. $(A \cup B)' \cap (E' \cup A) = \{9\}$

Order of Set Operations

DeMorgan's Laws for Set Theory If *A* and *B* are sets, then $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$.

Order of Set Operations

The Inclusion-Exclusion Principle If *A* and *B* are sets, then $n\left(A \cup B\right) = n(A) + n(B) - n(A \cap B)$.