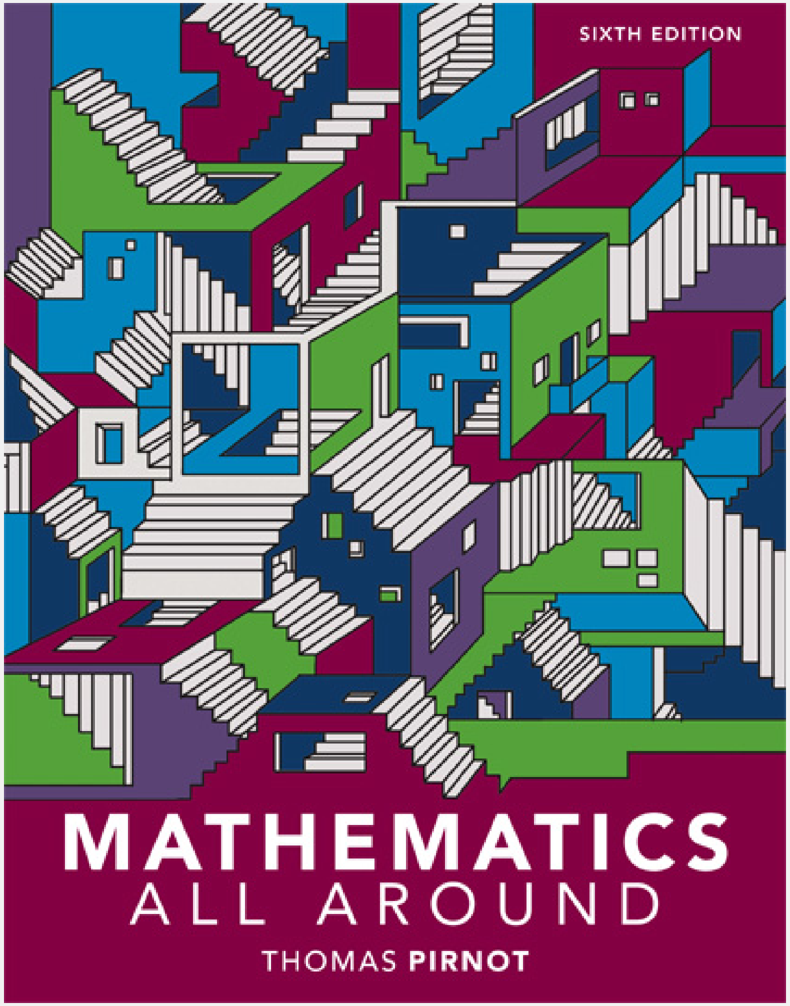
2

Set Theory



2.2 Comparing Sets

- Determine when sets are equal.
- Distinguish between the ideas of "equal" and "equivalent" sets.
- Explain the difference between subsets and proper subsets.
- Use Venn diagrams to illustrate set relationships.
- Determine the number of subsets of a given set.

Set Equality

Two sets A and B are **equal** if they have exactly the same members. In this case, we write $A = B$. If A and B are not equal, we write $A \neq B$.

Example: Set Equality

Which pairs of sets are equal?

a) {Facebook, Flickr, Twitter, Pinterest, Instagram, Vine} and {Pinterest, Flickr, Instagram, Twitter, Facebook, Vine, Flickr, Pinterest}

Notice that both sets contain *exactly* the same elements. Neither the order nor the repetition of elements is important; therefore, the two sets are equal.

Example: Set Equality (cont)

Which pairs of sets are equal?

b) $A = \{x : x$ is a citizen of the United States$\}$ and $B = \{y : y$ was born in the United States$\}$

Because Arnold Schwarzenegger is an element of set A, but is not an element of set B, the sets are not equal. Can you think of any other elements of A that are not in B?

Equivalent Sets

Sets A and B are **equivalent,** or in **one-to-one correspondence,** if $n(A) = n(B)$. Another way of saying this is that two sets are equivalent if they have the same number of elements.

Example: Distinguishing Between Equal and Equivalent Sets

Which of the following pairs of sets are equal? Equivalent?

a) {a, b, c, d} {d, b, a, c, d, c}

b) {1, 2, 3} {4, 5, 6}

Example: Distinguishing Between Equal and Equivalent Sets (cont)

Solution

a) {a, b, c, d} {d, b, a, c, d, c}

Recall that the <u>repetition of elements does not increase the number of members in a set</u>. So, we could omit the repetition and write these sets as {a, b, c, d} and {d, b, a, c}. These sets are equal because they contain the same elements, and because they have the same number of elements, they can be put in one-to-one correspondence.

Example: Distinguishing Between Equal and Equivalent Sets (cont)

Solution

b) {1, 2, 3} {4, 5, 6}

These sets are clearly *not equal* but *are equivalent* because they can be placed in one-to-one correspondence

Subset

The set A is a **subset** of the set B if every element of A is also an element of B. We indicate this relationship by writing $A \subseteq B$. If A is not a subset of B, then we write $A \not\subseteq B$.

Problem Solving

Strategy: The Analogies Principle*

Similarities in notation and terminology often reflect corresponding similarities in the ideas being represented. Because the notation for “is a subset of,” $\subseteq$, reminds us of the notation for “less than or equal to,” $\leq$, we might expect that both ideas share similar properties.

Example: Identify Subsets

Determine whether either set is a subset of the other.

a) $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$

b) A = {Viola Davis, Peter Dinklage, Neil Patrick Harris, Melissa McCarthy} and $E = \{x : x \text{ has won an Emmy}\}$

Solution

a) Every member of A is also in B so we can say $A \subseteq B$. Because there is an element of B that is not in A, we write $B \not\subseteq A$.

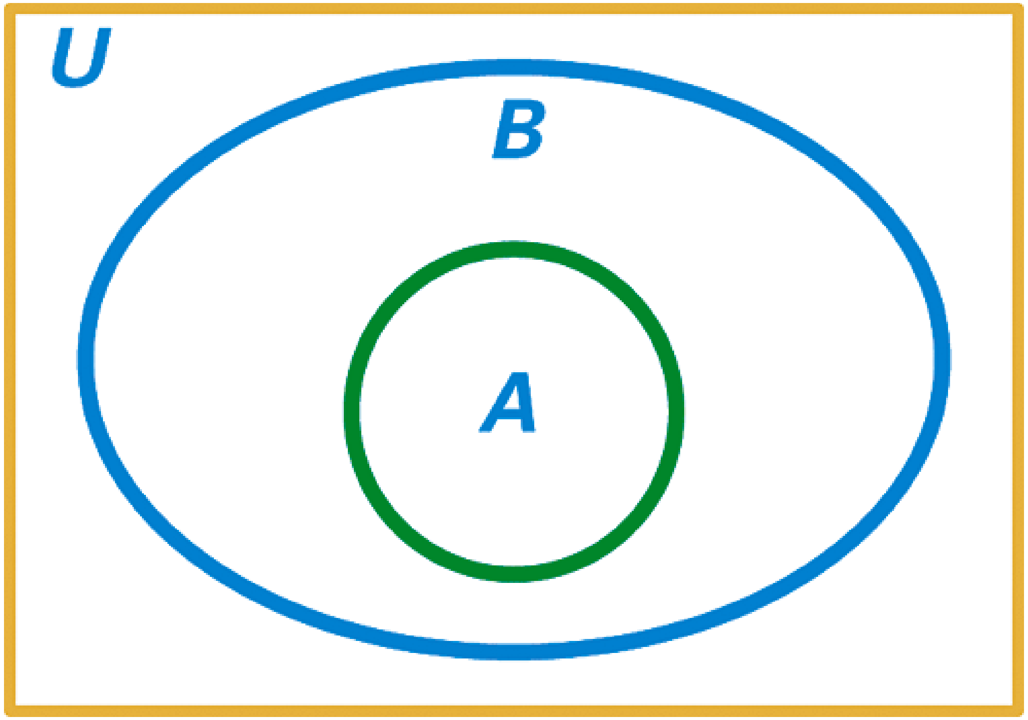
Example: Identify Subsets (cont)

b) A = {Viola Davis, Peter Dinklage, Neil Patrick Harris, Melissa McCarthy} and $E = \{x : x \text{ has won an Emmy}\}$

Because every member of set A has won an Emmy, we can say that $A \subseteq E$. However, many people have won Emmys who are not in set A. For example, Bill Murray won an Emmy in 2015 but is not in set A, so E is not a subset of A.

Venn Diagrams

A ***Venn diagram*** is used to visualize relationships among sets.



Proper Subset

The set A is a **proper subset** of the set B if $A \subseteq B$ but $A \neq B$. We write this as $A \subset B$. If A is not a proper subset of B, then we write $A \not\subset B$.

Example: Identifying Subsets

Consider the following table of medalist winners at the 2014 Winter Olympic Games in Sochi, Russia (next slide).

Assume that this set is the universal set, and define the following sets:

S = the set of slalom medalists

A = the set of U.S. athletes

G = the set of gold medal winners

N = the set of Norwegian athletes

B = the set of bronze medal winners

Example: Identifying Subsets (cont)

Which statements are true?

Athlete	Event	Medal	Country
Mathias Mayer	Slalom	Gold	Austria
David Wise	Halfpipe	Gold	United States
Tina Maze	Slalom	Gold	Slovenia
Henrik Kristoffersen	Slalom	Bronze	Norway
Hanna Kearney	Moguls	Bronze	United States
Juliya Dzhyma	Biathlon	Silver	Ukraine
Martin Sundby	Skiathlon	Bronze	Norway
Anna Haag	Relay	Gold	Sweden
Meryl Davis	Ice dancing	Gold	United States
Dmitry Japarov	Cross country	Silver	Russia

Example: Identifying Subsets (cont)

a) $A \subseteq G$

This is false because Kearney is a member of A but not a member of G.

Athlete	Event	Medal	Country
Mathias Mayer	Slalom	Gold	Austria
David Wise	Halfpipe	Gold	United States
Tina Maze	Slalom	Gold	Slovenia
Henrik Kristoffersen	Slalom	Bronze	Norway
Hanna Kearney	Moguls	Bronze	United States
Juliya Dzhyma	Biathlon	Silver	Ukraine
Martin Sundby	Skiathlon	Bronze	Norway
Anna Haag	Relay	Gold	Sweden
Meryl Davis	Ice dancing	Gold	United States
Dmitry Japarov	Cross country	Silver	Russia

S = the set of slalom medalists
A = the set of U.S. athletes
G = the set of gold medal winners
N = the set of Norwegian athletes
B = the set of bronze medal winners

b) $N \subseteq B$

This is true. Every member of N—namely, Kristoffersen and Sundby—is a member of B.

Example: Identifying Subsets (cont)

c) $N \subset B$

This is true. We already know that N is a subset of B because B contains Kearney, who is not in N, we can say $N \subset B$.

Athlete	Event	Medal	Country
Mathias Mayer	Slalom	Gold	Austria
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Henrik Kristoffersen	Slalom	Bronze	Norway
Hanna Kearney	Moguls	Bronze	United States
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THE NUMBER OF SUBSETS OF A SET

A set that has k elements has 2^k subsets.

As example, find the number of subsets in the set: {1, 2, 3, 4}

THE NUMBER OF SUBSETS OF A SET

A set that has *k* elements has 2^k subsets.

Size of Subset	Subsets of This Size	Number of Subsets of This Size
0	$\varnothing$	1
1	$\{1\}, \{2\}, \{3\}, \{4\}$	4
2	$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$	6
3	$\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$	4
4	$\{1, 2, 3, 4\}$	1
		Total $=$ 16