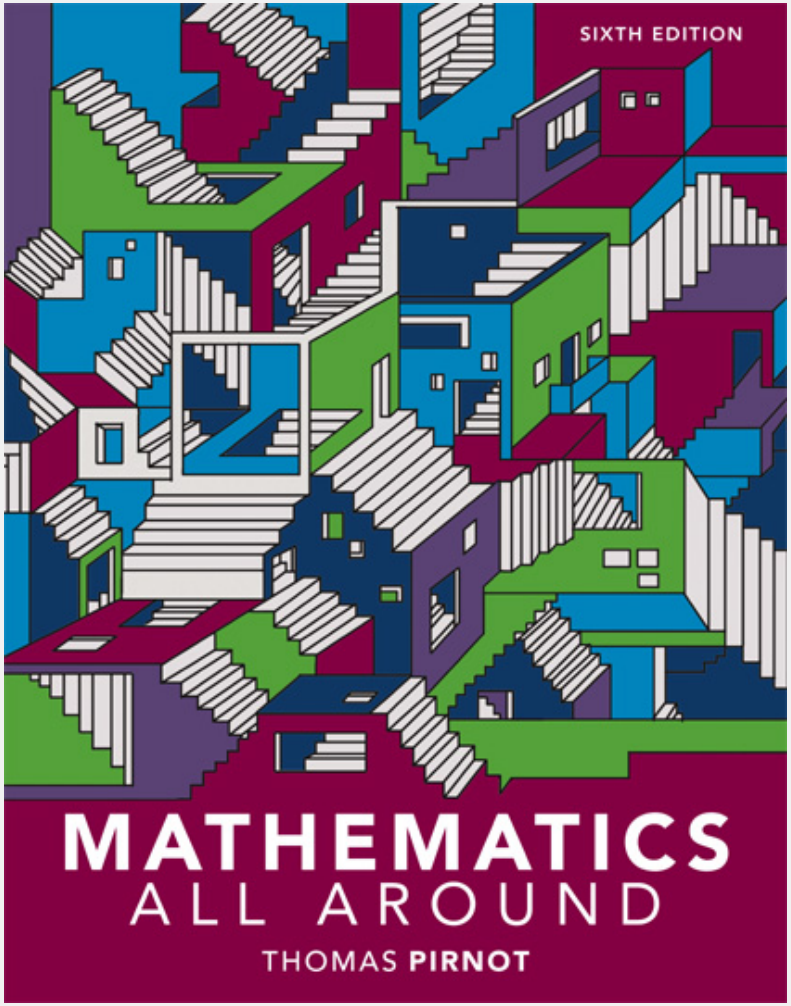
# 2

# Set Theory



# 2.1 The Language of Sets

- Specify sets using both listing and set-builder notation.
- Understand when sets are well-defined.
- Use the element symbol properly.
- Find the cardinal number of sets.

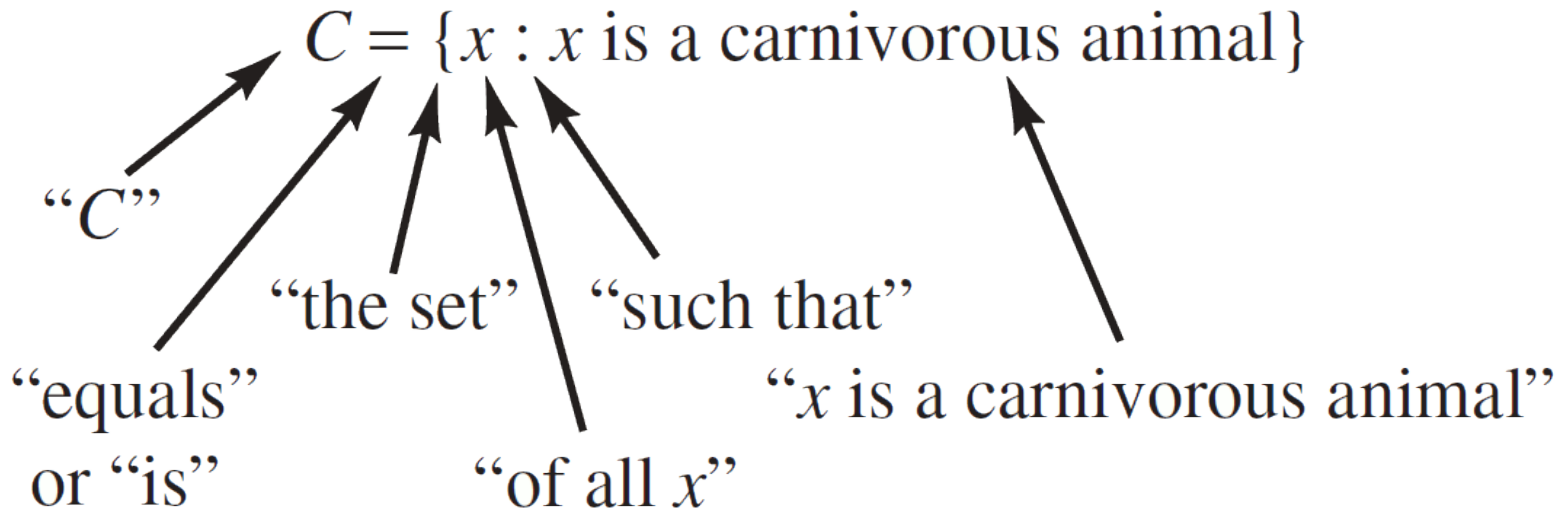
# Representing Sets

*Set* – collection of objects

*Element* – a member of a set

# Representing Sets

## Set-builder notation



# Representing Sets

A set is *well-defined* if we are able to tell whether any particular object is an element of that set.

# Example: Determining Whether a Set Is Well Defined

Which sets are well defined?

a) $A = \{x : x \text{ is a winner of an Academy Award}\}$

b) $T = \{x : x \text{ is tall}\}$

# Example: Determining Whether a Set Is Well Defined (cont)

Solution

a) $A = \{x : x \text{ is a winner of an Academy Award}\}$

This set is well defined because we can always determine whether or not a person belongs to set *A*. Leonardo DiCaprio, Felicity Jones, and Ethan Hawke are members of set *A*, but Hillary Clinton, Harry Potter, and Drake are not members of *A* because they have never won an Oscar.

# Example: Determining Whether a Set Is Well Defined (cont)

b) $T = \{x : x \text{ is tall}\}$

Whether or not a person belongs to this set is a matter of how we interpret *tall*; therefore, $T$ is not well defined. Can you think of one situation in which a person who is 6 feet tall would be considered tall and a different situation in which that same person would be considered short?

# Representing Sets

The set that contains no elements is called the **empty set** or **null set.** This set is labeled by the symbol ∅. Another notation for the empty set is { }.

# Example: Using Similar Notations Precisely

a) Does {∅} have the same meaning as ∅?

b) Do {∅} and {0} mean the same thing?

Solution

a) Note that {∅} is not the same as ∅. To make this more clear, you might think of a set as a paper bag that you might get at a supermarket. Then, the empty set ∅ corresponds to an empty bag, whereas the set {∅} could be visualized as one bag containing a second bag, which is empty.

# Example: Using Similar Notations Precisely (cont)

b) Do {∅} and {0} mean the same thing?

Similarly, {0} is not the same as {∅}. If we make bag drawings, then we see that {∅} corresponds to a bag containing an empty bag, whereas {0} corresponds to a bag containing the number zero.

# Representing Sets

The **universal set** is the set of all elements under consideration in a given discussion. We often denote the universal set by the capital letter *U*.

Consider female consumers living in the U.S. The universal set is

$$U = \{x : x \text{ is a female cosumer living in the U.S.}\}$$

# The Element Symbol

$\in$ means "is an element of"
$\notin$ means "is *not* an element of"

# Example: Using Set Element Notation

Replace the symbol # in each statement by either $\in$ or $\notin$.

a) 3 # {2, 3, 4, 5}

b) {5} # {2, 3, 4, 5}

c) Bill Gates # {$x$ : $x$ is a billionaire}

d) jogging # {$y$ : $y$ is an aerobic exercise}

e) the ace of hearts # {$f$ : $f$ is a face card in a standard 52-card deck}

# Example: Using Set Element Notation (cont)

Solution

a) $3 \in \{2, 3, 4, 5\}$

b) $\{5\} \notin \{2, 3, 4, 5\}$

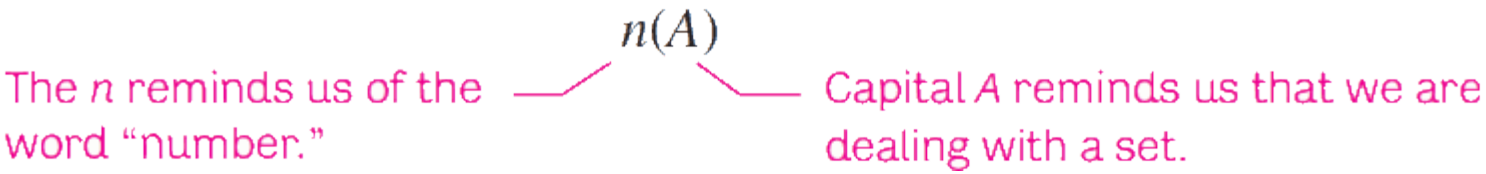
c) Bill Gates $\in \{x : x \text{ is a billionaire}\}$

d) jogging $\in \{y : y \text{ is an aerobic exercise}\}$

e) the ace of hearts $\notin \{f : f \text{ is a face card in a standard 52-card deck}\}$

# Cardinal Number

The number of elements in set *A* is called the **cardinal number** of set *A* and is denoted *n*(*A*). A set is **finite** if its cardinal number is a whole number. An **infinite** set is one that is not finite.



# Example: Finding the Cardinal Number of a Set

State whether each set is finite or infinite. If it is finite, state its cardinal number using $n(A)$ notation.

a) $P = \{x : x \text{ is a planet in our solar system}\}$

b) $N = \{1, 2, 3, ...\}$

c) $A = \{y : y \text{ is a person living in the United States who is not a citizen}\}$

d) $\varnothing$

e) $X = \{ \{1, 2, 3\}, \{1, 4, 5\}, \{3\} \}$

# Example: Finding the Cardinal Number of a Set (cont)

a) $P = \{x : x \text{ is a planet in our solar system}\}$

There are 8 planets. P is a finite set: $n(P) = 8$.

b) $N = \{1, 2, 3, ...\}$

The set of counting numbers is an infinite set.

c) $A = \{y : y \text{ is a person living in the United States who is not a citizen}\}$

There are a finite number of people living in the United States who are not citizens; however, we probably do not know $n(A)$.

# Example: Finding the Cardinal Number of a Set (cont)

d) $\varnothing$

The empty set has no elements, so it is a finite set. Thus, $n(\varnothing) = 0$.

e) $X = \{ \{1, 2, 3\}, \{1, 4, 5\}, \{3\} \}$

Set $X$ contains three objects: the set {1, 2, 3}, the set {1, 4, 5}, and the set {3}. Therefore, $n(X) = 3$.