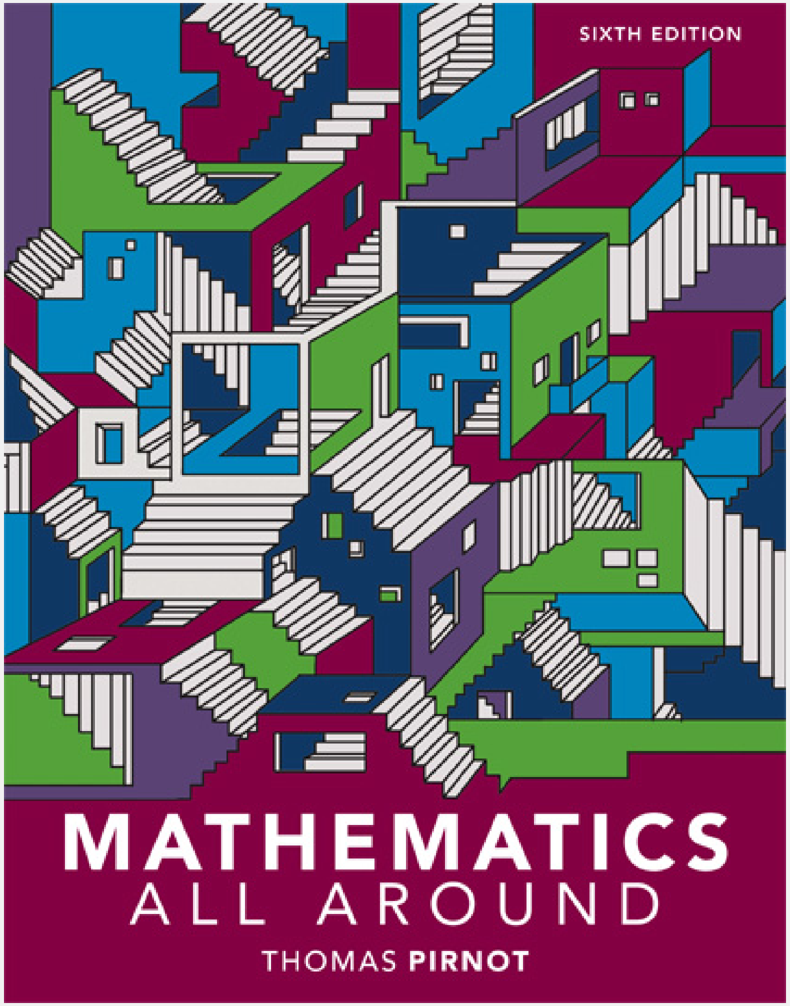
14.4

The Normal Distribution

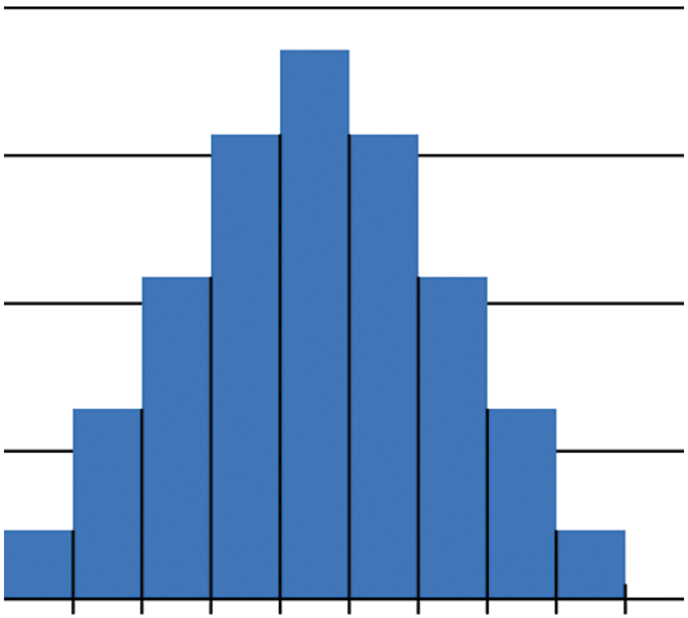


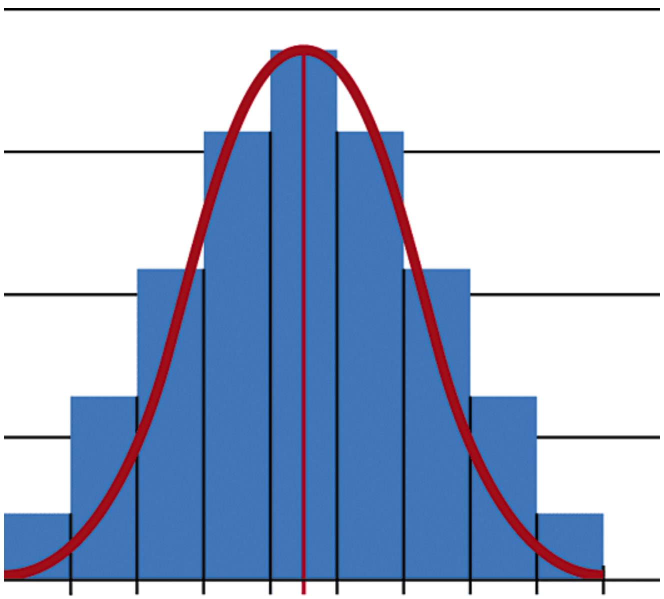
14.4 The Normal Distribution

- Describe the basic properties of the normal distribution.
- Relate the area under a normal curve to ***z***-scores.
- Make conversions between raw scores and ***z***-scores.
- Use the normal distribution to solve applied problems.

The Normal Distribution

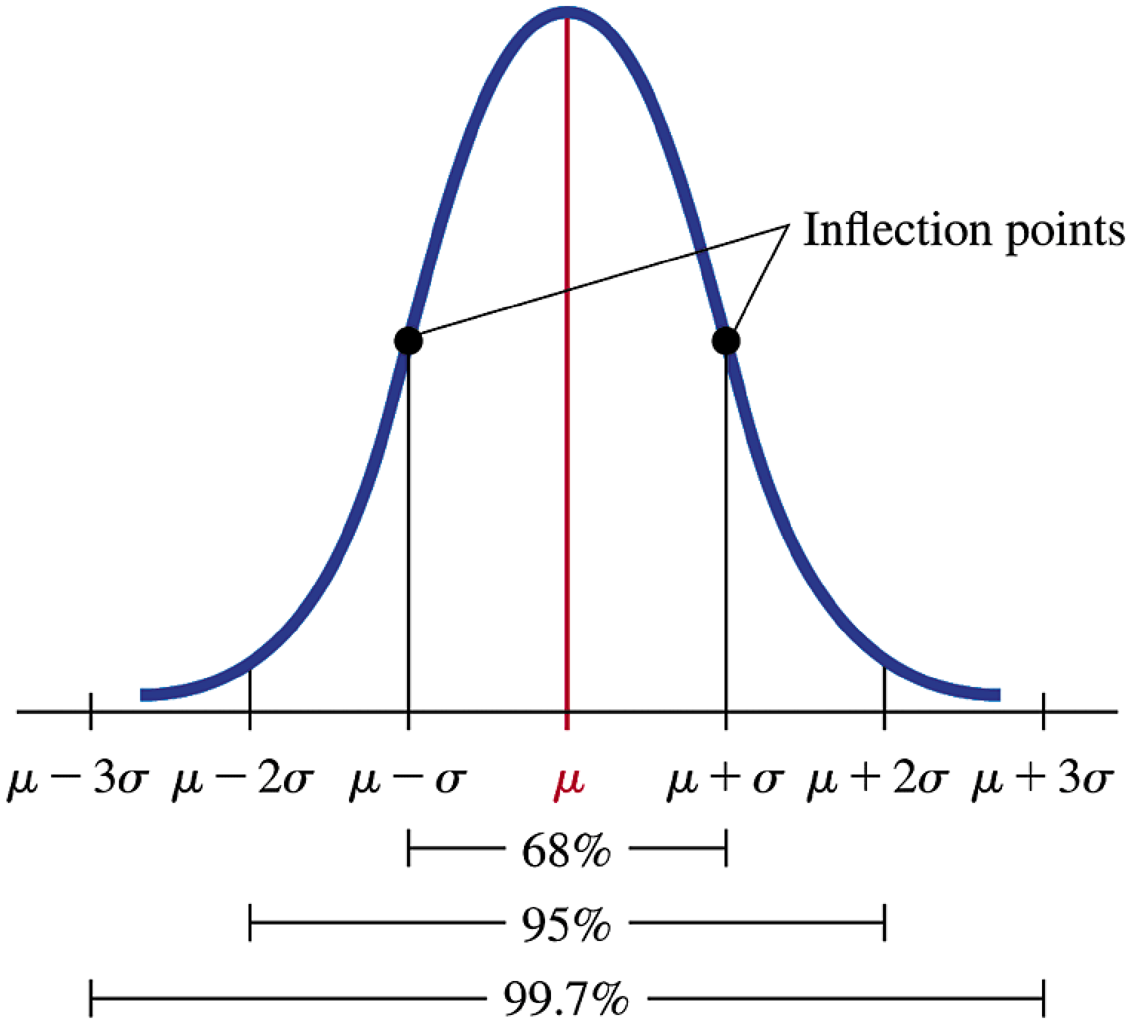
The *normal distribution* describes many real-life data sets. The histogram shown gives an idea of the shape of a normal distribution.





We represent the mean by μ and the standard deviation by σ.

The Normal Distribution



Properties of Normal Distribution

1. A normal curve is bell shaped.
2. The curve is symmetric with respect to its mean, where the curve reaches its highest point.
3. The mean, median, and mode of the distribution are the same.
4. The total area under the curve is 1.
5. Roughly 68% of the data values are within 1 standard deviation from the mean, 95% of the data values are within 2 standard deviations from the mean, and 99.7% of the data values are within 3 standard deviations from the mean.

Example: The Normal Distribution and Intelligence Tests

Suppose that the distribution of scores of 1,000 students who take a standardized intelligence test is a normal distribution. If the distribution's mean is 450 and its standard deviation is 25,

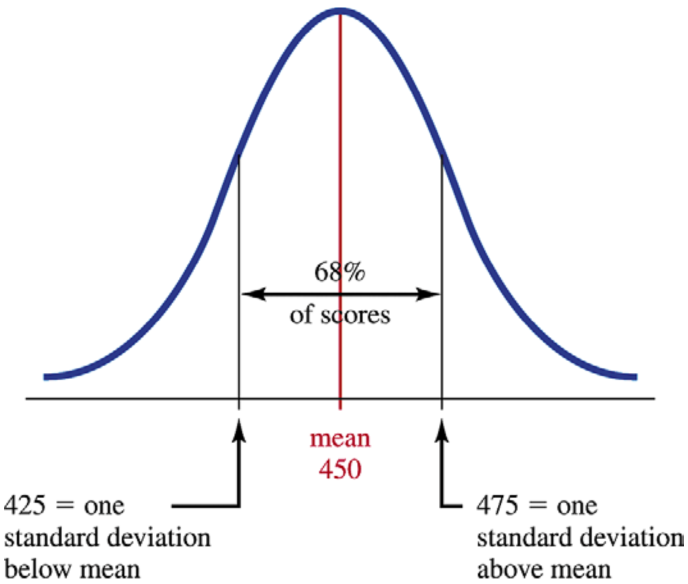
a) How many scores do we expect to fall between 425 and 475?

b) How many scores do we expect to fall above 500?

Example: The Normal Distribution and Intelligence Tests (cont)

Solution

The scores 425 and 475 are 1 standard deviation below and above the mean, respectively. Approximately 68% of the scores lie within 1 standard deviation of the mean.



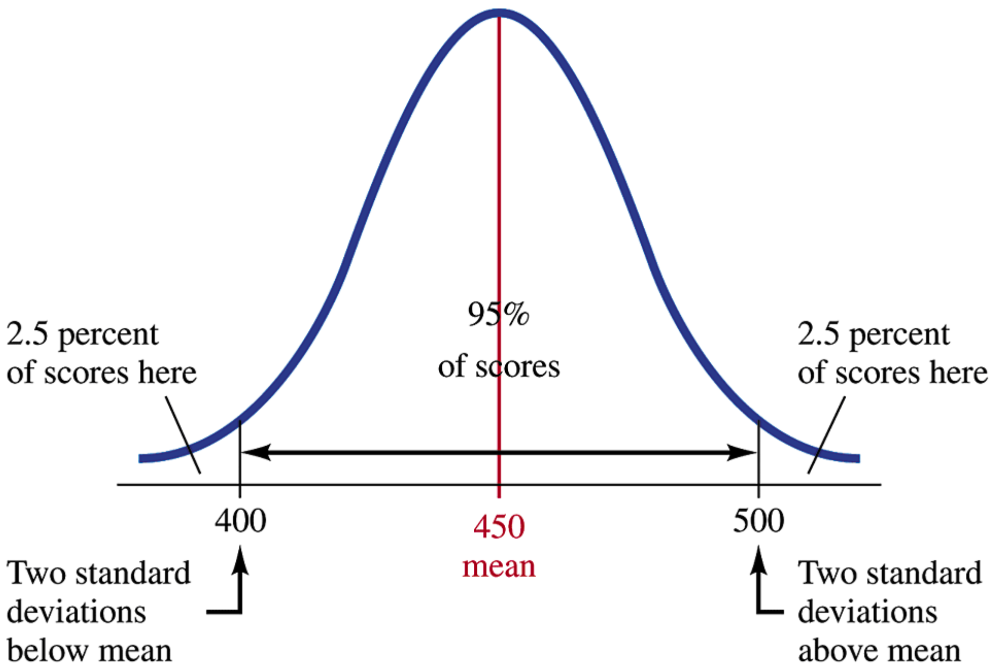
We expect about

$0.68 \times 1{,}000 = 680$ scores are in the range 425 to 475.

Example: The Normal Distribution and Intelligence Tests (cont)

b) We know 5% of the scores lie more than 2 standard deviations above or below the mean, so we expect to have 0.05 ÷ 2 = 0.025 of the scores to be above 500.

Multiplying by 1,000, we can expect that 0.025 × 1,000 = 25 scores to be above 500.



z-Scores

The *standard normal distribution* has a mean of 0 and a standard deviation of 1.

There are tables (see next slide) that give the area under this curve between the mean and a number called a z-score. A z-score represents the number of standard deviations a data value is from the mean.

For example, for a normal distribution with mean 450 and standard deviation 25, the value 500 is 2 standard deviations above the mean; that is, the value 500 corresponds to a z-score of 2.

Example: Finding Areas under the Standard Normal Curve

Use the table to find the percent of the data (area under the curve) that lies in the following regions for a standard normal distribution:

a) between $z = 0$ and $z = 1.3$

b) between $z = 1.5$ and $z = 2.1$

c) between $z = 0$ and $z = -1.83$

Converting Raw Scores to z-Scores

FORMULA FOR CONVERTING RAW SCORES TO z-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.

Example: Converting Raw Scores to z-Scores

In an effort to combat dog obesity, the Association for Pet Obesity Prevention has weighed a group of Basenjis (African barkless dogs) and found the distribution of their weights to be normally distributed with a mean of 20 pounds and a standard deviation of 3 pounds. Find the corresponding *z*-score for a dog in this group weighing:

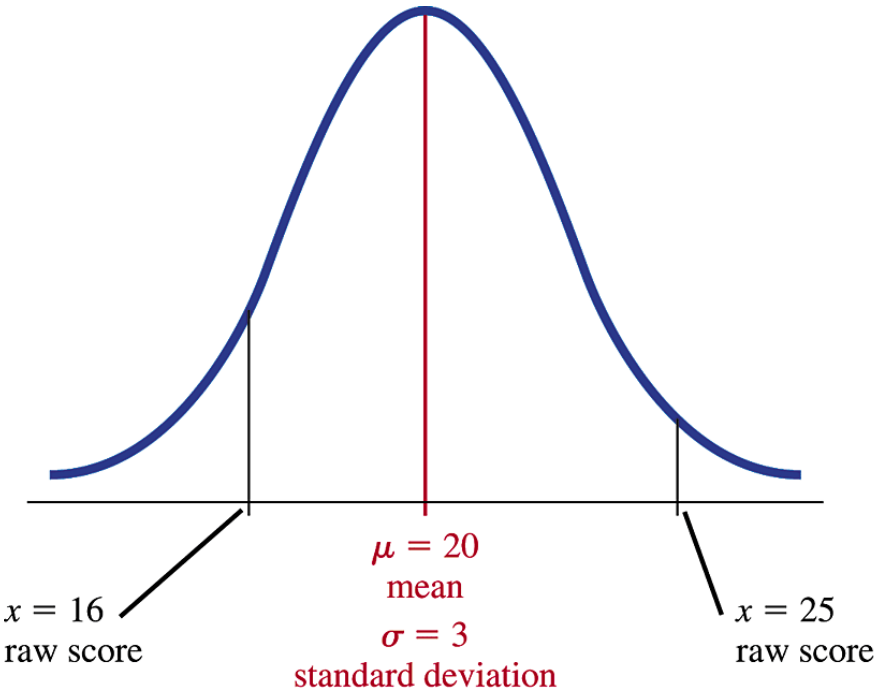
a) 25 pounds

b) 16 pounds

Example: Converting Raw Scores to z-Scores (cont)

Solution

a) We have
$$x = 25$$
$$\mu = 20$$
$$\sigma = 3.$$



We compute
$$z = \frac{x - \mu}{\sigma}$$
$$= \frac{25 - 20}{3}$$
$$= \frac{5}{3} = 1.67.$$

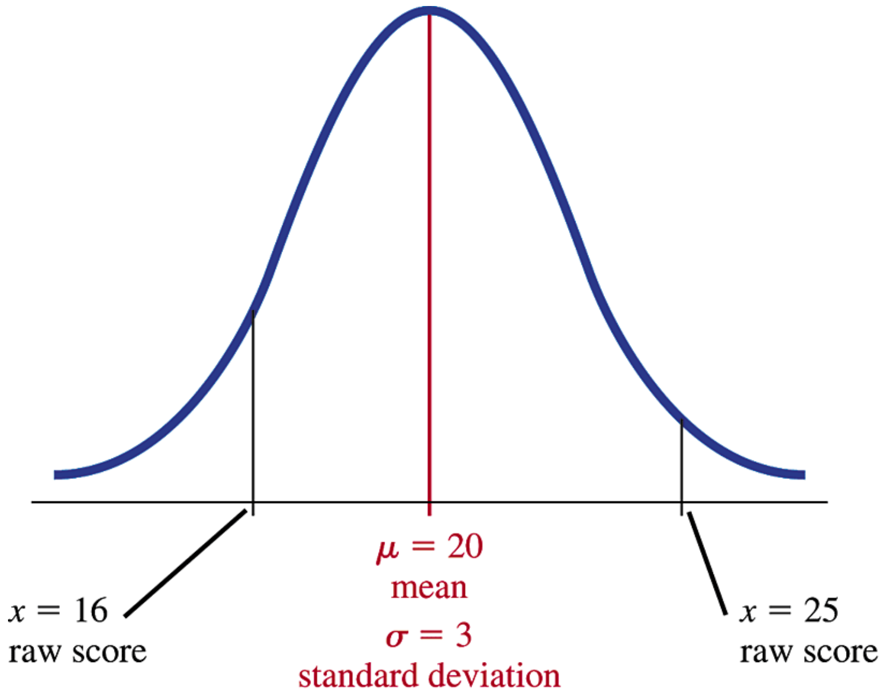
Example: Converting Raw Scores to z-Scores (cont)

Solution

b) We have $x = 16$

$\mu = 20$

$\sigma = 3.$



We compute $z = \dfrac{x - \mu}{\sigma}$

$$= \frac{16 - 20}{3}$$

$$= \frac{-4}{3} = -1.33.$$

Example: Interpreting the Significance of an Exam Score

Suppose that to qualify for a management training program offered by your employer you must score in the top 10% of those employees who take a standardized test. Assume that the distribution of scores is normal and you received a score of 72 on the test, which had a mean of 65 and a standard deviation of 4. What percent of those who took this test had a score below yours?

Example: Interpreting the Significance of an Exam Score (cont)

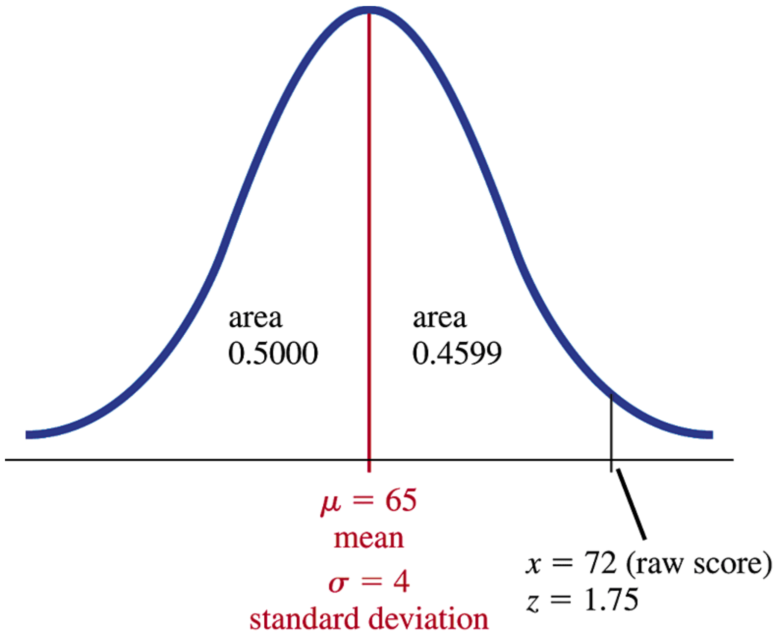
Solution

We first find the z-score that corresponds to 72.

$$z = \frac{72 - 65}{4} = \frac{7}{4} = 1.75$$

Example: Interpreting the Significance of an Exam Score (cont)

Using a table, we have that for $z = 1.75$. the area is 0.4599. Therefore, 45.99% of the scores fall between the mean and your score.



Example: Interpreting the Significance of an Exam Score (cont)

However, we must not forget the scores that fall below the mean. Because a normal curve is symmetric, another 50% of the scores fall below the mean. So, there are
50% + 45.99% = 95.99% of the scores below yours. Congratulations! You qualify for the program.

Example: Using z-Scores to Compare Data

Consider the following information:

Ty Cobb hit .420 in 1911.

Ted Williams hit .406 in 1941.

In the 1910s, the mean batting average was .266 and the standard deviation was .0371.

In the 1940s, the mean batting average was .267 and the standard deviation was .0326.

Assume that the batting averages were normally distributed in both of these decades. Use *z*-scores to determine which batter was ranked higher in relationship to his contemporaries.

Example: Using z-Scores to Compare Data (cont)

Solution

Ty Cobb’s average of .420 corresponded to a z-score of

$$\frac{.420-.266}{.0371}=\frac{.154}{.0371}\approx 4.15.$$

Ted William’s average of .460 corresponded to a z-score of

$$\frac{.406-.267}{.0326}=\frac{.139}{.0326}\approx 4.26.$$

We see that when we use the standard deviation to compare each hitter with his contemporaries, Ted Williams was ranked as the better hitter.

14.3

Turn in 14.4.53, 14.4.54, and 14.4.57 for next class.