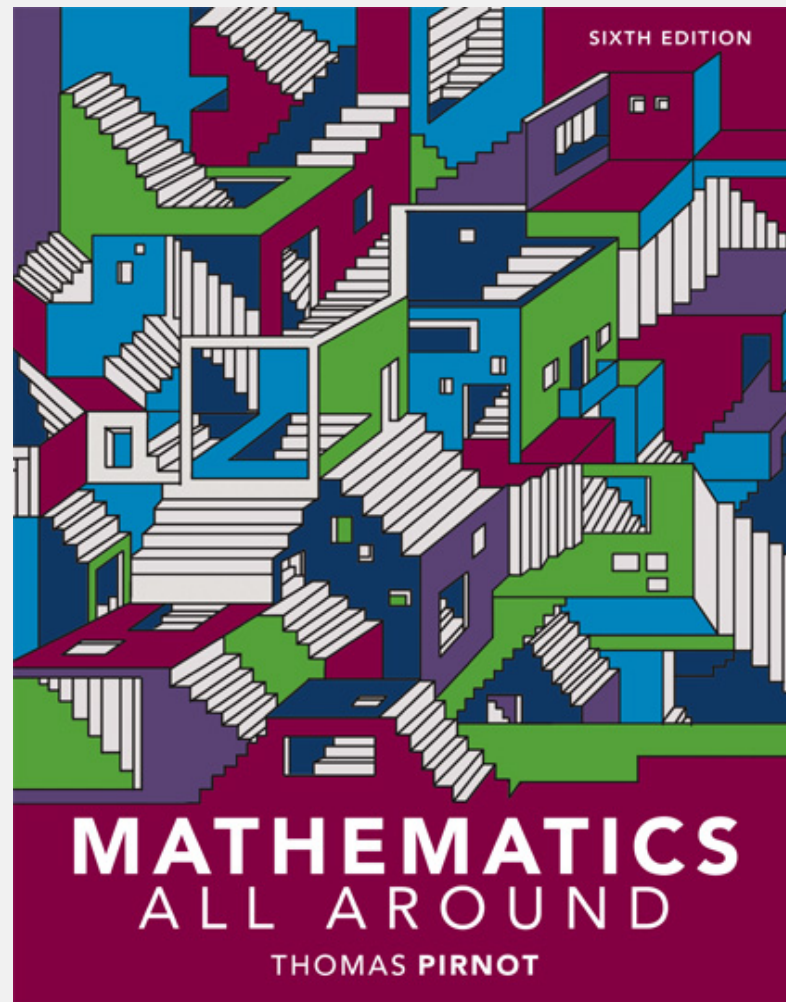# 14.3

Descriptive Statistics

Measures of Dispersion

14.3 Measures of Dispersion

- Compute the range of a data set.

- Explain how the standard deviation measures the spread of a distribution.

- Use the coefficient of variation to compare the relative variability of different distributions.

The Range of a Data Set

The **range** of a data set is the difference between the largest and smallest data values in the set.

Example: Comparing Heights

Find the range of the heights of the people listed in the table.

Person	Height	Height in Inches
Robert Wadlow (world's tallest person)	8 feet, 11 inches	107 inches
Tim Tebow	6 feet, 3 inches	75 inches
Madge Bester (world's shortest woman)	2 feet, 2 inches	26 inches
LeBron James	6 feet, 8 inches	80 inches

Example: Comparing Heights (cont)

| Person	Height	Height in Inches
Robert Wadlow (world's tallest person)	8 feet, 11 inches	107 inches
Tim Tebow	6 feet, 3 inches	75 inches
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LeBron James	6 feet, 8 inches	80 inches

Solution

The range of these data is range = largest data value − smallest data value

= 107 − 26 = 81 inches = 6 feet, 9 inches.

Standard Deviation

DEFINITION If x is a data value in a set whose mean is $\overline{x}$, then $x - \overline{x}$ is called x's *deviation from the mean.*

Data Value, x	Deviation from Mean, $x - \overline{x}$
16	−1
14	−3
12	−5
21	4
22	5
Total	0
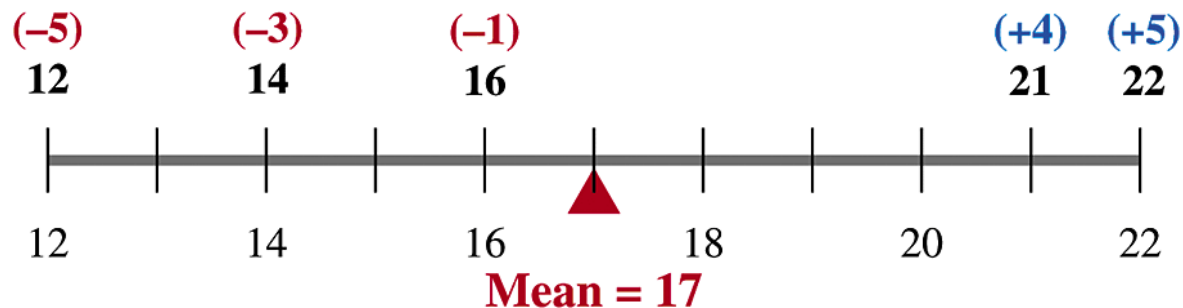
Standard Deviation

DEFINITION We denote the **standard deviation** of a *sample* of n data values by s,[†] which is defined as follows:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}.$$

Data Value, x	Deviation from Mean, $x - \bar{x}$	Deviation Squared, $(x - \bar{x})^2$
16	-1	1
14	-3	9
12	-5	25
21	4	16
22	5	25

$$\frac{1 + 9 + 25 + 16 + 25}{5 - 1} = \frac{76}{4} = 19, \ s = \sqrt{19} \approx 4.36$$

Standard Deviation

COMPUTING THE STANDARD DEVIATION To compute the standard deviation for a sample consisting of n data values, do the following:

1. Compute the mean of the data set; call it $\bar{x}$.

2. Find $(x - \bar{x})^2$ for each score x in the data set.

3. Add the squares found in step 2 and divide this sum by $n - 1$; that is, find

$$\frac{\sum (x - \bar{x})^2}{n - 1},$$

which is called the **variance**.

4. Compute the square root of the number found in step 3.

Example: Calculating the Standard Deviation of the Cost of Plane Flights

While planning for spring break, suppose you went to Orbitz.com to find the cheapest flight from Philadelphia to Orlando and found the following prices:

$195, $213, $208, $219, $210, $215.

Find the standard deviation of this data set.

Example: Calculating the Standard Deviation of the Cost of Plane Flights (cont)

Solution

Step 1:

$$\text{Mean} = \frac{195 + 213 + 208 + 219 + 210 + 215}{6} = \frac{1,260}{6} = 210$$

Example: Calculating the Standard Deviation of the Cost of Plane Flights (cont)

Step 2: We calculate the squares of the deviations of the data values from the mean.

Price, x	Deviation from Mean, $x - \bar{x}$	Deviation Squared, $(x - \bar{x})^2$
195	$195 - 210 = -15$	$(-15)^2 = 225$
213	$213 - 210 = 3$	$3^2 = 9$
208	$208 - 210 = -2$	$(-2)^2 = 4$
219	$219 - 210 = 9$	$9^2 = 81$
210	$210 - 210 = 0$	$0^2 = 0$
215	$215 - 210 = 5$	$5^2 = 25$
		$\sum(x - \bar{x})^2 = 344$

Example: Calculating the Standard Deviation of the Cost of Plane Flights (cont)

Step 3: We average these squared deviations to get

$$\frac{225 + 9 + 4 + 81 + 0 + 25}{6 - 1} = \frac{344}{5} = 68.8.$$

Step 4: Taking the square root of the result in step 3, we get the standard deviation

$$s = \sqrt{68.8} \approx 8.29.$$

Example: Using a Frequency Table to Compute the Standard Deviation

Anya is considering investing in APPeal, a software company that specializes in smartphone applications. She intends to compute the standard deviation of its recent prices to measure how steady its stock price has been recently. The following are the closing prices for the stock for the past 20 trading sessions:

37, 39, 39, 40, 40, 38, 38, 39, 40, 41,

41, 39, 41, 42, 42, 44, 39, 40, 40, 41

What is the standard deviation for this data set?

Example: Using a Frequency Table to Compute the Standard Deviation (cont)

Solution

The sum of the closing prices is 800.

$$\text{Mean} = \frac{800}{20} = 40$$

Example: Using a Frequency Table to Compute the Standard Deviation (cont)

We create a table with values that will facilitate computing the standard deviation.

Closing Price, x	Frequency, f	Product, $x \cdot f$	Deviation, $(x - 40)$	Deviation Squared, $(x - 40)^2$	Product, $(x - 40)^2 \cdot f$
37	1	37	−3	9	9
38	2	76	−2	4	8
39	5	195	−1	1	5
40	5	200	0	0	0
41	4	164	1	1	4
42	2	84	2	4	8
44	1	44	4	16	16
	$\Sigma f = 20$	$\Sigma (x \cdot f) = 800$			$\Sigma (x - 40)^2 \cdot f = 50$

Standard Deviation:

$$s = \sqrt{\frac{\sum (x - 40)^2 \cdot f}{n - 1}} = \sqrt{\frac{50}{19}} \approx 1.62$$
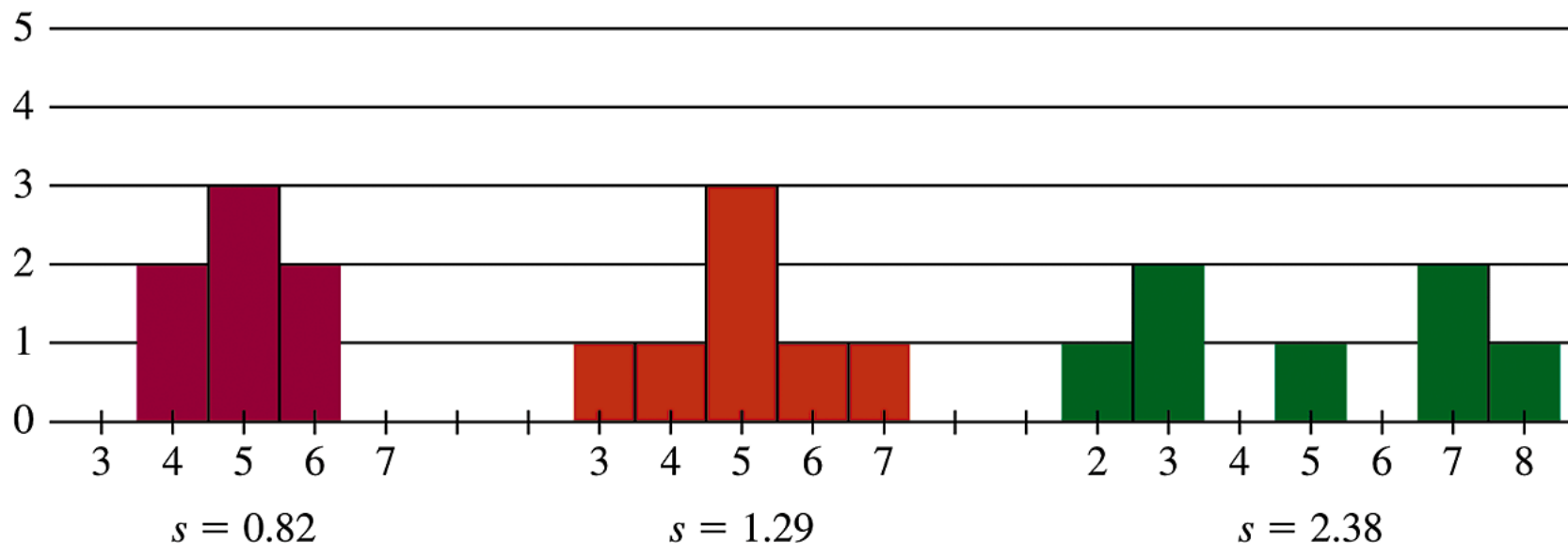
Standard Deviation

FORMULA FOR COMPUTING THE SAMPLE STANDARD DEVIATION FOR A FREQUENCY DISTRIBUTION We calculate the standard deviation, s, of a sample that is given as a frequency distribution as follows:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2 \cdot f}{n - 1}},$$

where $\bar{x}$ is the mean of the distribution, f is the frequency of data value x, and $n = \Sigma f$ is the number of data values in the distribution.

Standard Deviation

$s = 0.82$ $s = 1.29$ $s = 2.38$

All three distributions have a mean and median of 5; however, as the spread of the distribution increases, so does the standard deviation.

The Coefficient of Variation or Relative Standard Deviation

DEFINITION For a set of data with mean $\bar{x}$ and standard deviation s, we define the **coefficient of variation**, denoted by *CV*, as

$$CV = \frac{s}{\bar{x}} \cdot 100\%.$$

Example: Using the Coefficient of Variation to Compare Data

Find the mean and standard deviation of the data. Then find the coefficient of variation and determine whether Facebook or Twitter had more consistent closing stock prices during the first six months of 2016.

Month	Facebook	Twitter
January	118.78	15.02
February	118.81	15.22
March	117.58	14.62
April	114.10	16.55
May	106.92	18.12
June	112.21	16.80

Example: Using the Coefficient of Variation to Compare Data

Solution

You could use a calculator, app, or online calculator to verify that the mean closing price for Facebook was $114.73 with a standard deviation of $4.67. For Twitter, the mean was $16.06 with standard deviation $1.33. The coefficient of variation for Facebook is

$$\frac{4.67}{114.73} \cdot 100\% = 4.07\%.$$

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Example: Using the Coefficient of Variation to Compare Data

For Twitter, the coefficient of variation is

$$\frac{1.33}{16.06} \cdot 100\% = 8.28\%.$$

Thus, using the coefficient of variation, we find that there was more relative variability for Twitter than for Facebook. The closing stock prices for Facebook were, relatively speaking, more consistent. This is because the mean closing price for Facebook ($114.73) is more than seven times the mean for Twitter ($16.06), but the standard deviation for Facebook is less than four times that of Twitter.