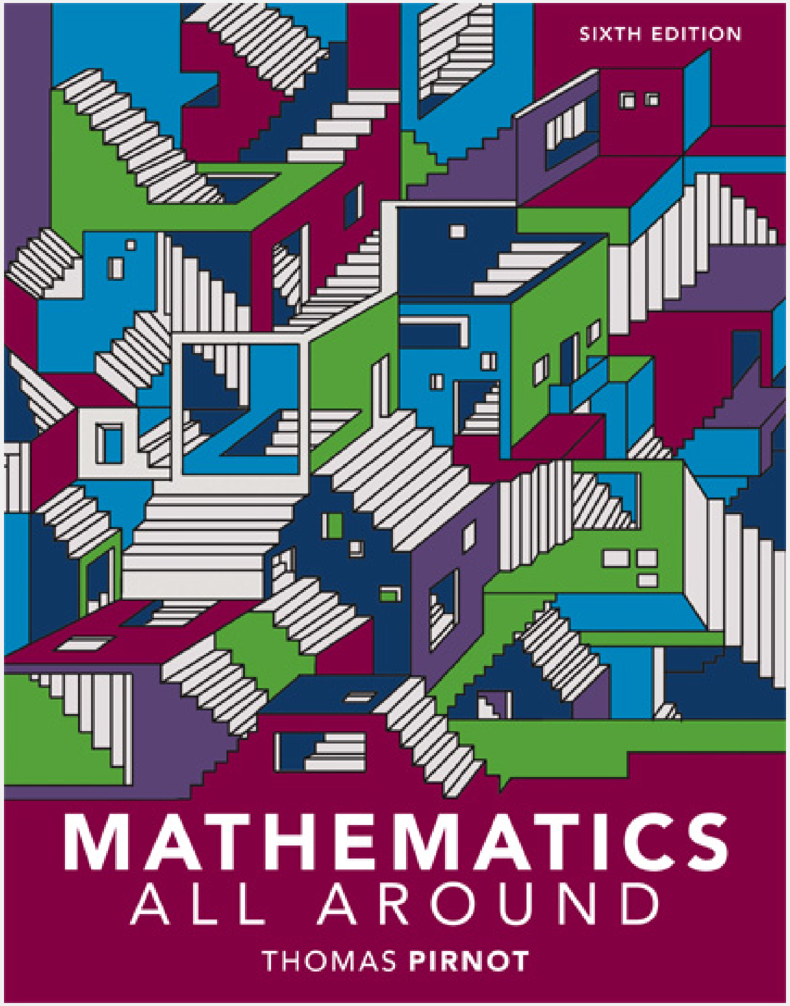
13.3

Probability: Conditional and Intersection of Events



13.3 Conditional Probability and Intersection of Events

- Be able to compute conditional probabilities.
- Calculate the probability of the intersection of two events.
- Use probability trees to compute conditional probabilities.
- Be able to determine the difference when events are dependent and independent events.

Conditional Probability

When we compute the probability of event *F* assuming that the event *E* has already occurred, we call this the **conditional probability** of *F* given *E*. We denote this probability as $P(F|E)$. We read $P(F|E)$ as "the probability of *F* given that *E* has occurred," or in a quicker way, "the probability of *F* given *E*."

Special Rule for Computing $P(F|E)$ by Counting

If E and F are events in a sample space with equally likely outcomes, then

$$P(F \mid E) = \frac{n(E \cap F)}{n(E)}.$$

Example: Computing Conditional Probability by Counting

Assume that we roll two dice and the total showing is greater than nine. What is the probability that the total is odd?

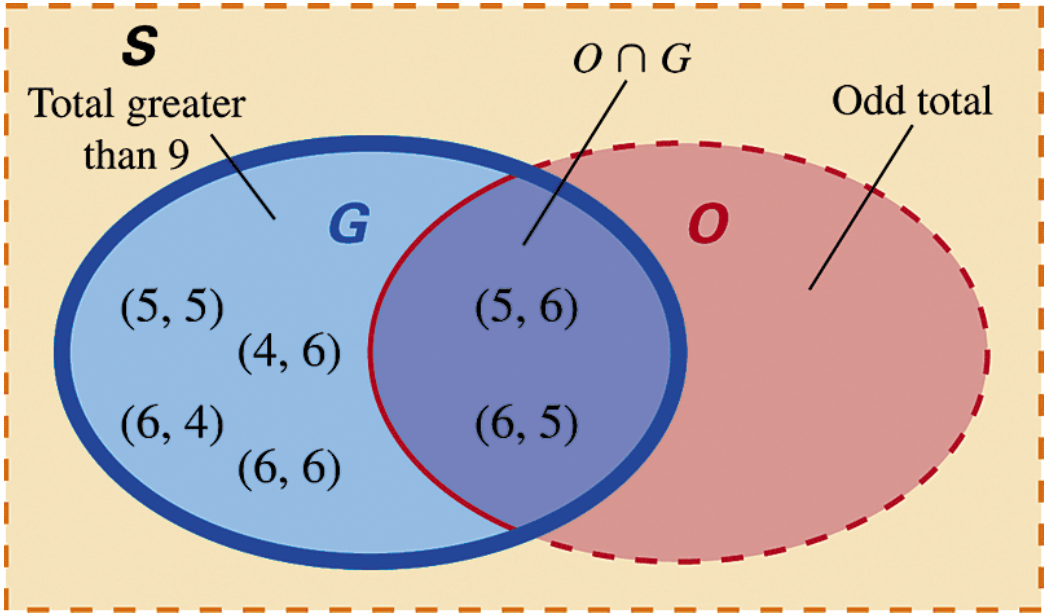
Solution

This sample space has 36 equally likely outcomes. We will let G be the event “we roll a total greater than nine” and let O be the event “the total is odd.” Therefore,

$$G = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}.$$

Example: Computing Conditional Probability by Counting (cont)

We now seek all pairs that give an odd total – the diagram below shows that there are two.



$$P(O \mid G) = \frac{n(O \cap G)}{N(G)}$$

$$= \frac{2}{6} = \frac{1}{3}$$

General Rule for Computing $P(F|E)$

If E and F are events in a sample space not necessarily equally likely,

then

$$P(F \mid E) = \frac{P(E \cap F)}{P(E)}.$$

Example: Using the General Rule for Computing Conditional Probability

The state bureau of labor statistics conducted a survey of college graduates comparing starting salaries to majors. The survey results are listed in the table on the next slide. If we select a graduate who was offered between $40,001 and $45,000, what is the probability that the student has a degree in the health fields?

Example: Using the General Rule for Computing Conditional Probability (cont)

Major	$30,000 and Below	$30,001 to $35,000	$35,001 to $40,000	$40,001 to $45,000	Above $45,000	Totals (%)
Liberal arts	6*	10	9	1	1	27
Science	2	4	10	2	2	20
Social sciences	3	6	7	1	1	18
Health fields	1	1	8	3	1	14
Technology	0	2	7	8	4	21
Totals (%)	12	23	41	15	9	100

* These numbers are percentages.

Example: Using the General Rule for Computing Conditional Probability (cont)

Solution

Let *R* be the event "graduate received a starting salary between \$40,001 and \$45,000" and *H* the event "student has a degree in the health fields." We want to find *P*(*H* | *R*).

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Liberal arts	6	10	9	1	1	27
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TABLE 13.7 The columns we want to ignore are darkened. $P(R)$ $P(H \cap R)$

Example: Using the General Rule for Computing Conditional Probability (cont)

$$P(H \mid R) = \frac{P(H \cap R)}{P(R)} = \frac{0.03}{0.15} = 0.20$$

If a graduate earns a starting salary between $40,001 and $45,000, then the probability that the person is in the health fields is 0.20, or 20%.

The Intersection of Events

Rule for Computing the Probability of the Intersection of Events

If *E* and *F* are two events, then

$$P(E \cap F) = P(E) \cdot P(F \mid E).$$

Example: Estimating Your Grade in a Class

Assume that for your literature final your professor has written questions on 10 assigned readings on cards and you are to randomly select two cards and write an essay on each. If you have read 8 of the 10 readings, what is your probability of getting two questions that you can answer? (We will assume that if you've done a reading, then you can answer the question about that reading; otherwise, you can't answer the question.)

Example: Estimating Your Grade in a Class (cont)

Solution

We can think of this event as the intersection of two events *A* and *B*, where

A is "you can answer the first question";

B is "you can answer the second question."

By the rule we just stated, you need to calculate

$$P(A \cap B) = P(A) \cdot P(B \mid A).$$

Example: Estimating Your Grade in a Class (cont)

Calculate the probabilities.

$$P(A) = \frac{8}{10} \qquad P(B \mid A) = \frac{7}{9}$$

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B \mid A) \\ &= \frac{8}{10} \cdot \frac{7}{9} \\ &= \frac{56}{90} \approx 0.62 \end{aligned}$$

Probability Trees

Using Trees to Calculate Probabilities

We can represent an experiment that happens in stages with a tree whose branches represent the outcomes of the experiment. We calculate the probability of an outcome by multiplying the probabilities found along the branch representing that outcome. We will call these trees **probability trees**.

Example: Drug Testing

Assume that you are working for a company that has a mandatory drug-testing policy. It is estimated that 2% of the employees use a certain drug, and the company is giving a test that is 99% accurate in identifying users of this drug. What is the probability that if an employee is identified by this test as a drug user, the person in fact is not a drug user?

i.e. a ***false Positive***!

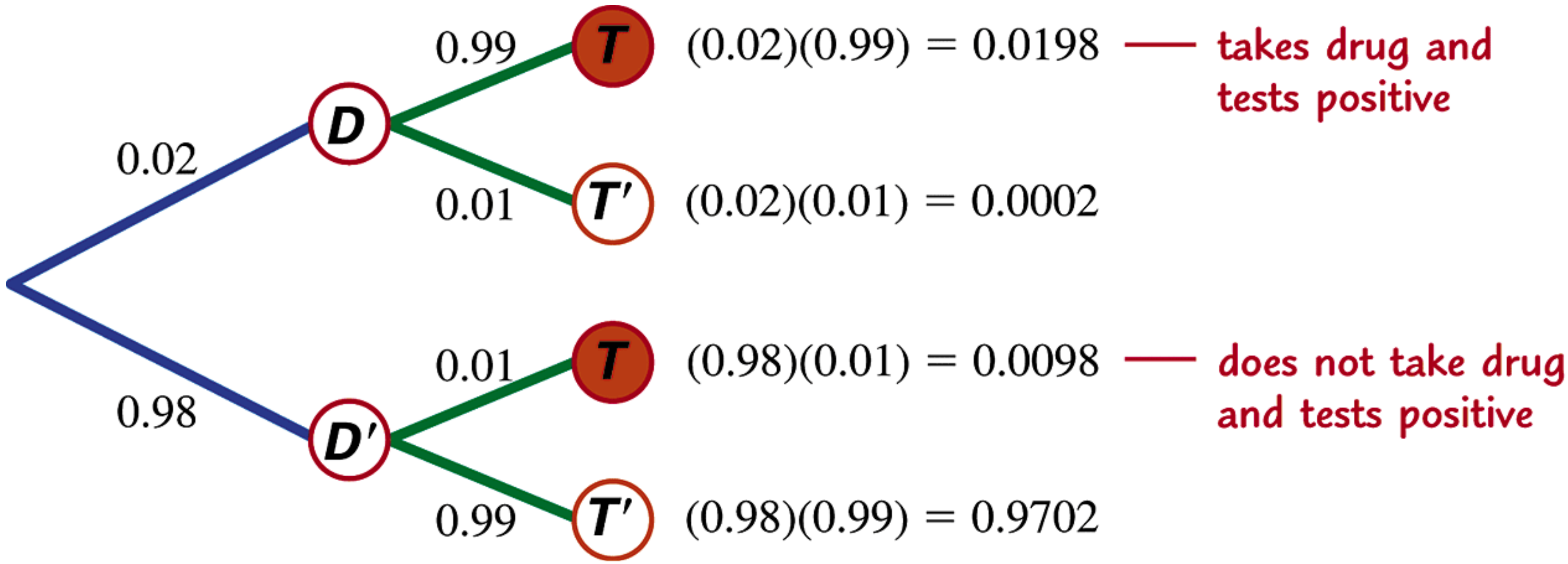
Example: Drug Testing (cont)

Solution

Let D be the event "the person is a drug user" and let T be the event "the person tests positive for the drug." We are asking, then, if we are given that the person tests positive, what is the probability that the person does not use the drug? Realize that the complement of D—namely, D'—is the event "the person does not use the drug." So, we are asking for the conditional probability $P(D'|T)$.

Example: Drug Testing (cont)

$P(D'|T)$



$$P(T) = (0.02)(0.99) + (0.98)(0.01)$$

$$P(D' \cap T) = (0.98)(0.01)$$

Example: Drug Testing (cont)

The probability that an innocent person will test positive for the drug is

$$P(D'|T) = \frac{P(D' \cap T)}{P(T)} = \frac{(0.98)(0.01)}{(0.02)(0.99) + (0.98)(0.01)}$$

$$= \frac{0.0098}{0.0296} \approx 0.331.$$

Dependent and Independent Events

Events E and F are **independent** events if $P(F|E) = P(F)$.

If $P(F|E) \neq P(F)$, then E and F are dependent.

Example: Determining Whether Events Are Independent or Dependent

Assume we roll a red and a green die. Are the events *F*, "a five shows on the red die," and *G*, "the total showing on the dice is greater than 10," independent or dependent?

Solution

The three outcomes (5, 6), (6, 5), and (6, 6) give a total greater than 10, so

$$P(G) = \frac{3}{36} = \frac{1}{12}$$

Example: Determining Whether Events Are Independent or Dependent (cont)

We have $F = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$, and $G \cap F = \{(5, 6)\}$.

$$P(G \mid F) = \frac{P(G \cap F)}{P(F)} = \frac{1/36}{6/36} = \frac{1}{6}$$

If $P(G|F) \neq P(G)$, the events are dependent.