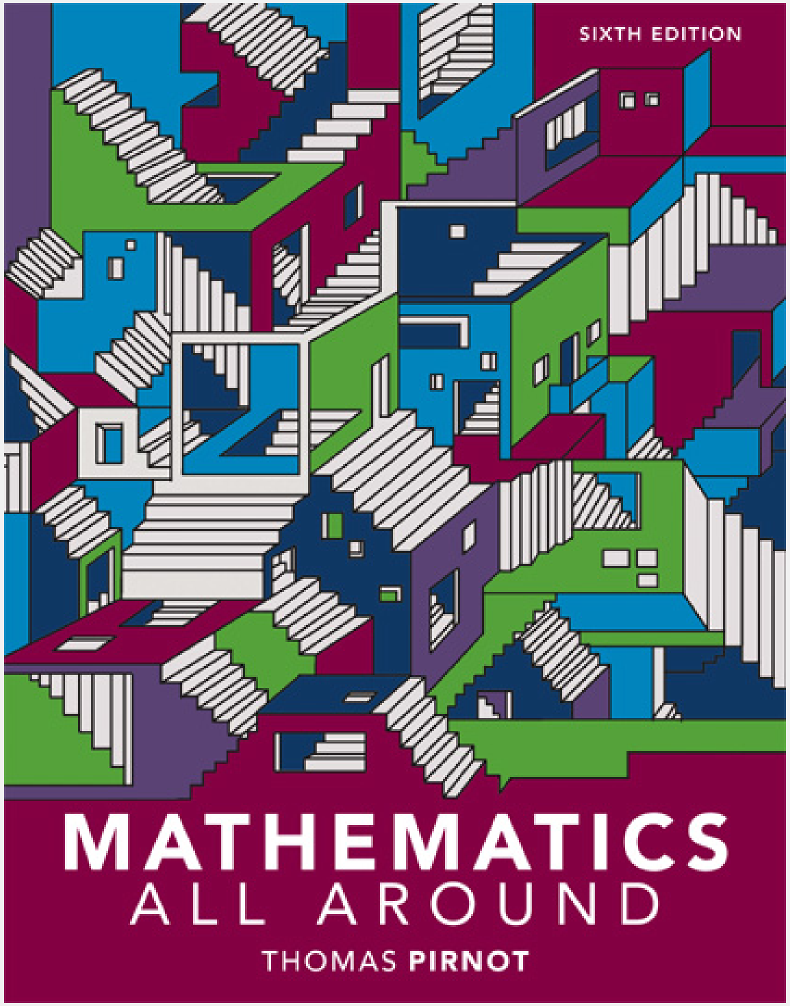
13.2

Probability: Complements and Union of Events



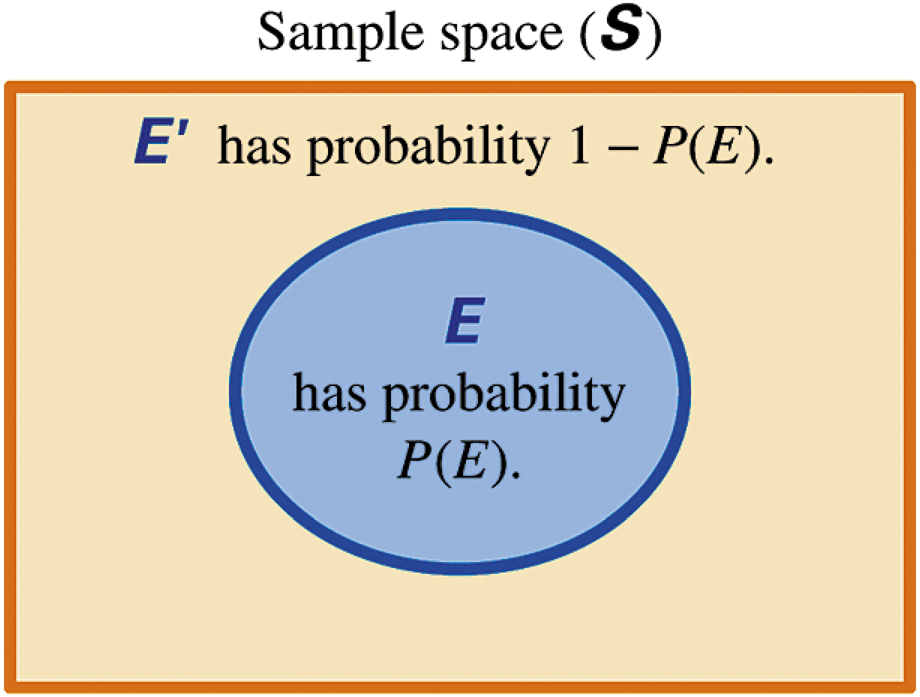
13.2 Complements and Unions of Events

- Use the relationship between the probability of an event and its complement to simplify problems.
- Calculate the probability of the union of two events.
- Use complement and union formulas to compute the probability of an event.

Complements of Events

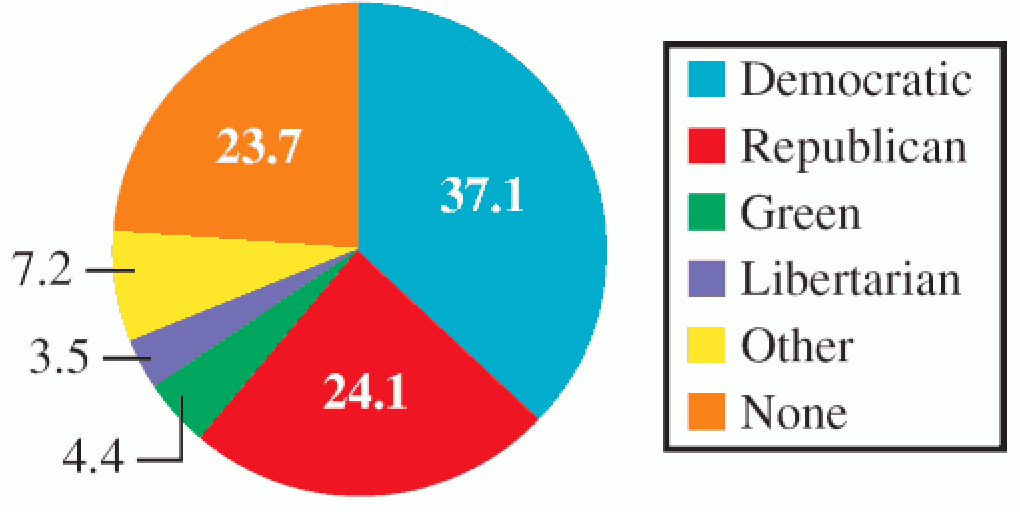
Computing the Probability of the Complement of an Event

If E is an event, then $P(E') = 1 – P(E)$.



Example: Using the Complement Formula to Study Voter Affiliation

The accompanying graph shows how a group of first-time voters are classified according to their party affiliation. If we randomly select a person from this group, what is the probability that the person has a party affiliation?



Percent of voters according to party affiliation

Example: Using the Complement Formula to Study Voter Affiliation (cont)

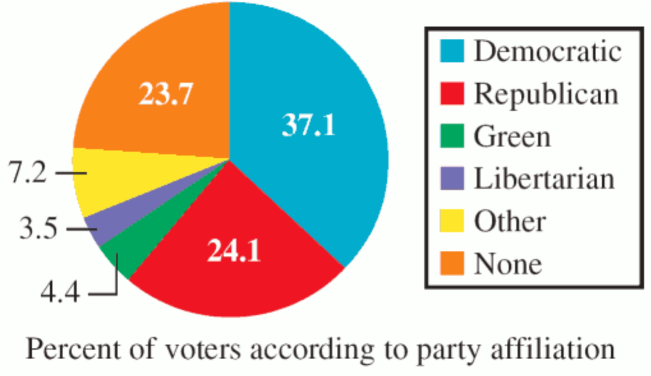
Solution

Let *A* be the event that the person we select has some party affiliation. Rather than compute the probability of this event, it is simpler to calculate the probability of *A*, which is the event that the person we select has no party affiliation. We illustrate this situation in the figure on the next slide. It is important to remember that the total probability available in this sample space is 1.

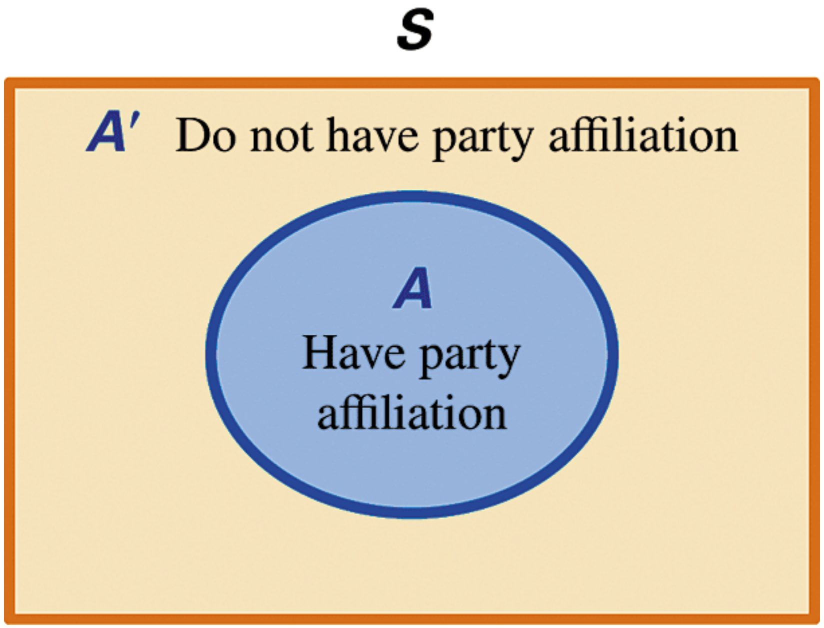
Example: Using the Complement Formula to Study Voter Affiliation (cont)

Because 23.7% have no party affiliation, the probability of selecting such a person is 0.237.

$$P(A) = P(S) - P(A')$$
$$= 1 - P(A')$$
$$= 1 - 0.237 = 0.763$$

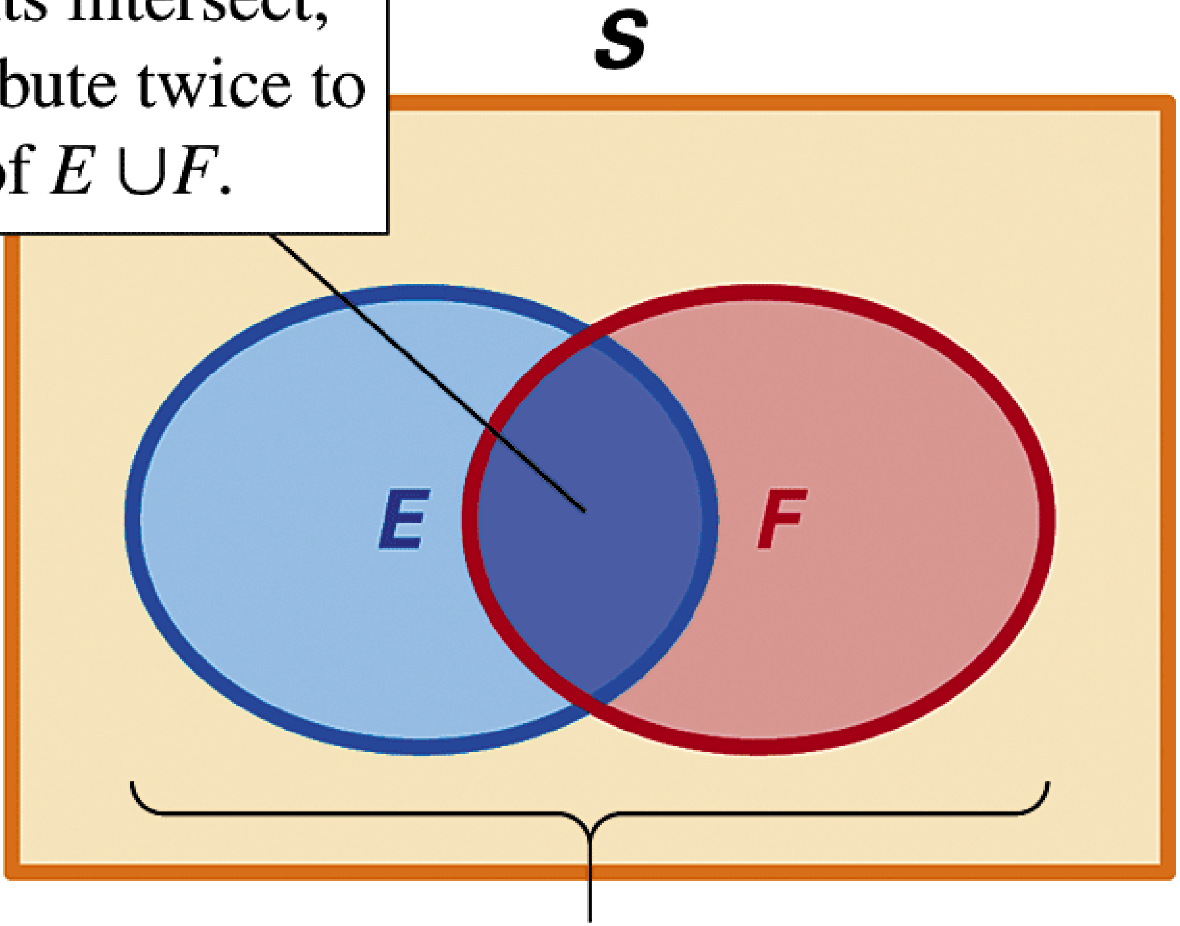


Percent of voters according to party affiliation



Union of Events

Where two events intersect, outcomes contribute twice to the probability of $E \cup F$.



Union of E and F

Union of Events

Rule for Computing the Probability of a Union of Two Events

If *E* and *F* are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If *E* and *F* have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \varnothing,$ the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$

Example: Finding the Probability of the Union of Two Events

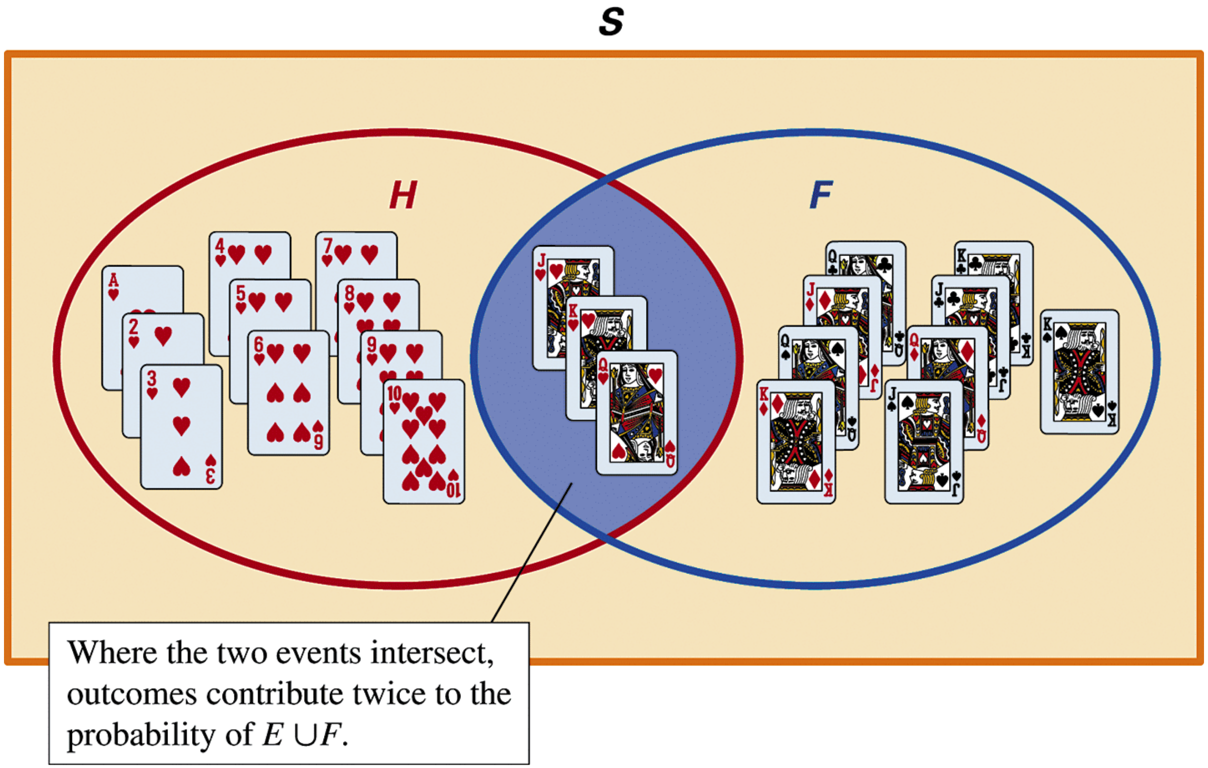
If we select a single card from a standard 52-card deck, what is the probability that we draw either a heart or a face card?

Solution

Let *H* be the event “draw a heart” and *F* be the event “draw a face card.” We are looking for $P(H \cup F)$.

Example: Finding the Probability of the Union of Two Events (cont)

There are 13 hearts, 12 face cards, and 3 cards that are both hearts and face cards.



Where the two events intersect, outcomes contribute twice to the probability of $E \cup F$.

Example: Finding the Probability of the Union of Two Events (cont)

Therefore,

$$P(H \cup F) = P(H) + P(F) - P(E \cap F)$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$= \frac{22}{52} = \frac{11}{26}$$

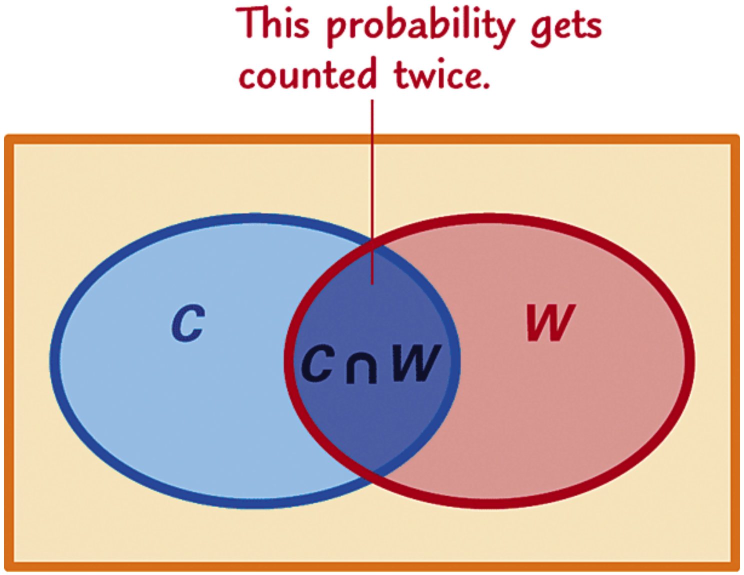
Example: Using Algebra to Find a Missing Probability

A magazine conducted a survey of readers age 18 to 25 regarding their health concerns. The editors will use this information to choose topics relevant to their readers. The survey found that 35% of the readers were concerned with improving their cardiovascular fitness and 55% wanted to lose weight. Also, the survey found that 70% are concerned with either improving their cardiovascular fitness or losing weight. If the editors randomly select one of those surveyed to profile in a feature article, what is the probability that the person is concerned with both improving cardiovascular fitness *and* losing weight?

Example: Using Algebra to Find a Missing Probability (cont)

Solution

Let *C* be the event “the person wants to improve cardiovascular fitness” and *W* be the event “the person wishes to lose weight.” We need to find $P(C \cap W)$.



This probability gets counted twice.

We have $P(C) = 0.35$, $P(W) = 0.55$, and $P(C \cup W) = 0.70$.

Example: Using Algebra to Find a Missing Probability (cont)

We have
$$P(C \cup W) = P(C) + P(W) - P(C \cap W)$$
$$0.70 = 0.35 + 0.55 - ?$$

$$P(C \cap W) = 0.35 + 0.55 - 0.70$$
$$= 0.20$$

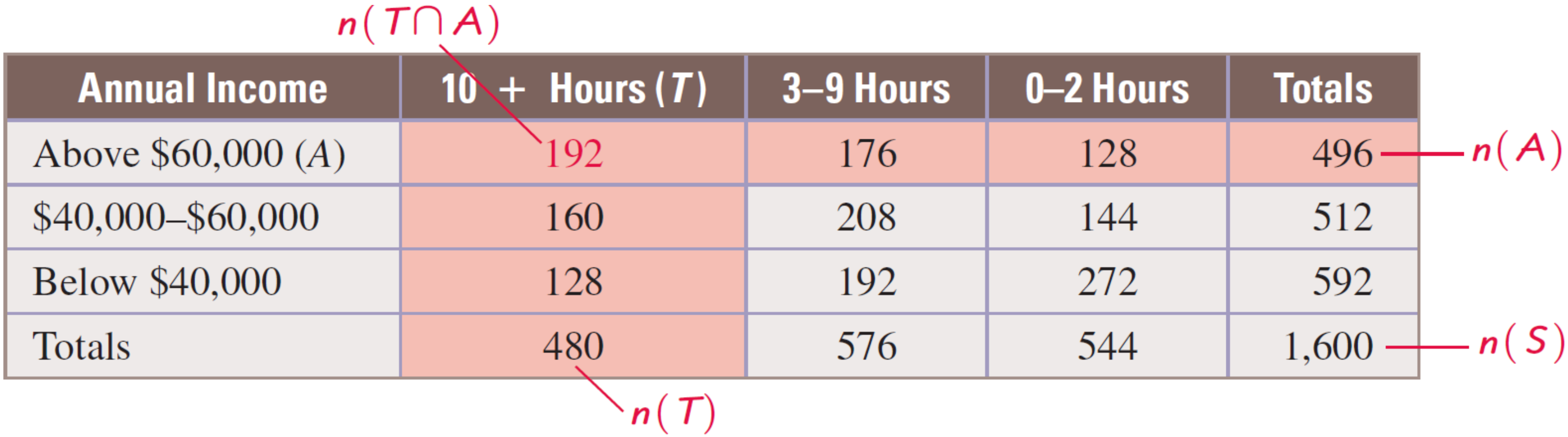
This means that there is a 20% chance that the person chosen will be interested in both improving cardiovascular fitness and losing weight.

Example: Finding the Probability of the Complement of the Union of Two Events

A survey of 1600 consumers comparing the amount of time they spend shopping on the Internet per month with their annual income produced the results shown on the next slide.

Assume that these results are representative of all consumers. If we select a consumer randomly, what is the probability that the consumer neither shops on the Internet 10 or more hours per month nor has an annual income above $60,000?

Example: Finding the Probability of the Complement of the Union of Two Events



Annual Income	10 + Hours (*T*)	3–9 Hours	0–2 Hours	Totals
Above $60,000 (*A*)	192	176	128	496
$40,000–$60,000	160	208	144	512
Below $40,000	128	192	272	592
Totals	480	576	544	1,600

Let *T* be the event “the consumer selected spends 10 or more hours per month shopping on the Internet.” Let *A* be the event “the consumer selected has an annual income above $60,000.”

Example: Finding the Probability of the Complement of the Union of Two Events (cont)

From the table we get
$n(T) = 192 + 160 + 128 = 480$ and
$n(A) = 192 + 176 + 128 = 496.$
With $n(S)$ = 1,600, we may compute the probabilities below.

$$P(T) = \frac{n(T)}{n(S)} = \frac{480}{1{,}600} = 0.30$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{496}{1{,}600} = 0.31$$

Example: Finding the Probability of the Complement of the Union of Two Events (cont)

$$P(T \cap A) = \frac{n(T \cap A)}{n(S)} = \frac{192}{1,600} = 0.12$$

$$\begin{aligned} P\left((T \cup A)'\right) &= 1 - P(T \cup A) = 1 - [P(T) + P(A) - P(T \cap A)] \\ &= 1 - [0.30 + 0.31 - 0.12] \\ &= 1 - 0.49 \\ &= 0.51 \end{aligned}$$

This means that if we select a consumer randomly, there is a 51% chance that the consumer neither spends 10 or more hours per month shopping on the Internet nor has a yearly income above $60,000.