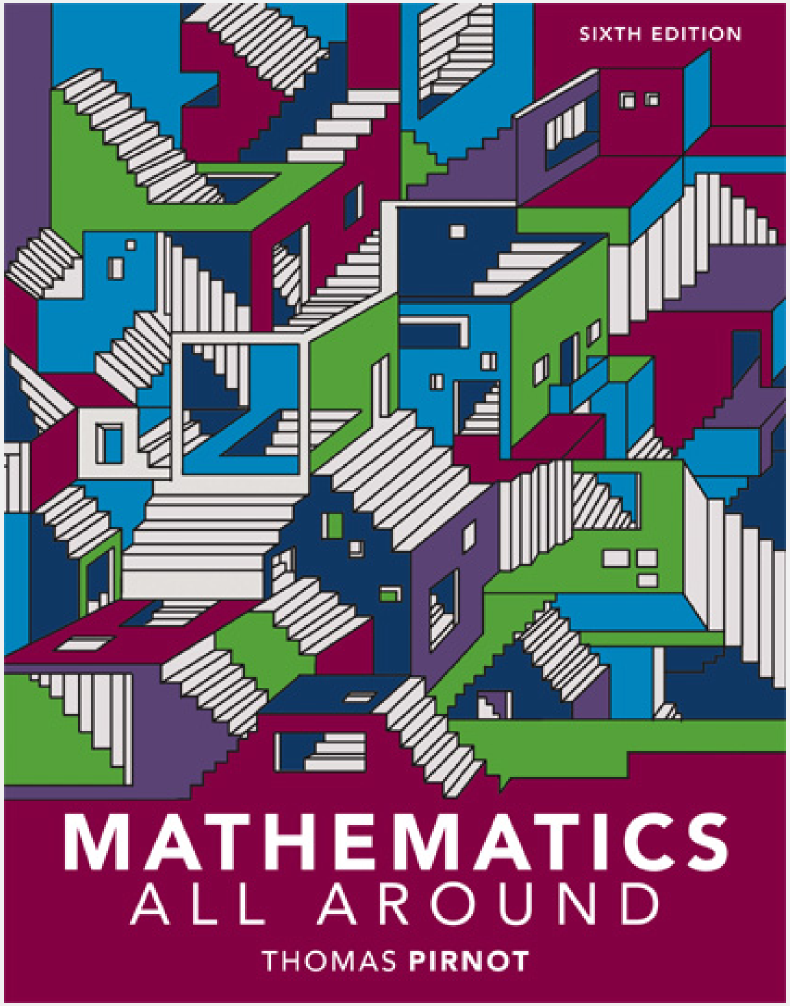
13.1

Probability



13.1 The Basics of Probability Theory

- Calculate probabilities by counting outcomes in a sample space.
- Use counting formulas to compute probabilities.
- Perform computations using the relationship between probability and odds.
- Use probability theory to investigate genetic diseases.

Sample Spaces and Events

Random phenomena are occurrences that vary from day-to-day and case-to-case.

Although we never know exactly how a random phenomenon will turn out there tends to be a pattern over many occurrences thus, we can often calculate a number called a *probability* that it will occur in a certain way.

e.g. tossing a coin, rolling dice, drawing a card

Sample Spaces and Events

An **experiment** is any observation of a random phenomenon preformed under controlled conditions to test the validity of a hypothesis.

- The different possible results of the experiment are called **outcomes**.
  - The set of all possible outcomes for an experiment is called a **sample space**.

Example: Finding Sample Spaces (cont)

Solution

a) We select an iPhone from a production line and determine whether it is defective.

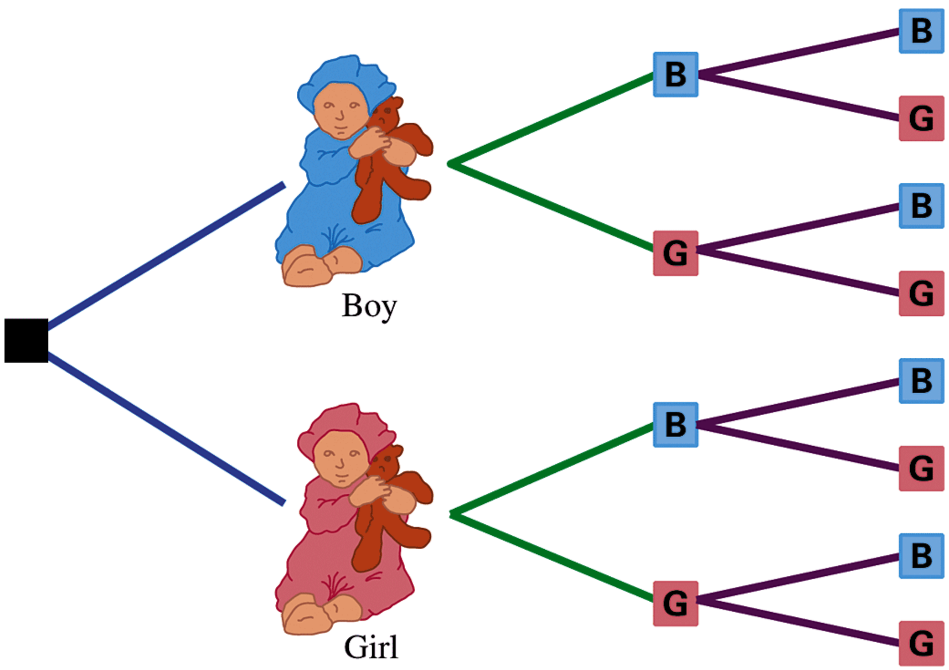
The sample space is {defective, nondefective}.

Example: Finding Sample Spaces (cont)

b) Three children are born to a family and we note the birth order with respect to gender.

We use a tree diagram to help find the sample space.

{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg}



Example: Finding Sample Spaces (cont)

c) We roll two dice and observe the pair of numbers showing on the top faces.

Try to systemically list all the results…

Example: Finding Sample Spaces (cont)

c) The sample space for this experiment consists of the following 36 pairs:

$$
\begin{array}{llllll}
\{(1, 1), & (1, 2), & (1, 3), & (1, 4), & (1, 5), & (1, 6), \\
(2, 1), & (2, 2), & (2, 3), & (2, 4), & (2, 5), & (2, 6), \\
(3, 1), & (3, 2), & (3, 3), & (3, 4), & (3, 5), & (3, 6), \\
(4, 1), & (4, 2), & (4, 3), & (4, 4), & (4, 5), & (4, 6), \\
(5, 1), & (5, 2), & (5, 3), & (5, 4), & (5, 5), & (5, 6), \\
(6, 1), & (6, 2), & (6, 3), & (6, 4), & (6, 5), & (6, 6)\}
\end{array}
$$

Sample Spaces and Events

In probability theory, an **event** is a subset of the sample space.

e.g. in a family of three children has two girls and one boy.

Example: Describing Events as Subsets

Write each event as a subset of the sample space.

a) A head occurs when we flip a single coin.

b) Two girls and one boy are born to a family.

c) A total of five occurs when we roll a pair of dice.

Solution

a) The set {head} is the event.

Example: Describing Events as Subsets (cont)

b) Two girls and one boy are born to a family.

Noting that the boy can be the first, second, or third child, the event is {ggb, gbg, bgg}.

c) A total of five occurs when we roll a pair of dice.

The following set shows how we can roll a total of five on two dice: {(1, 4), (2, 3), (3, 2), (4, 1)}.

Sample Spaces and Events

The **probability of an outcome** in a sample space is a number between 0 and 1 inclusive.

The sum of the probabilities of all the outcomes in the sample space must be 1.

The **probability of an event** E, written $P(E)$, is defined as the sum of the probabilities of the outcomes that make up E.

Sample Spaces and Events

Empirical Assignment of Probabilities

If *E* is an event and we perform an experiment several times, then we estimate the probability of *E* as follows:

$$P(E) = \frac{\text{the number of times } E \text{ occurs}}{\text{the number of times the experiment is performed}}.$$

This ratio is sometimes called the *relative frequency* of *E*. (recall 14.1!)

Example: Using Empirical Information to Assign Probabilities

A pharmaceutical company is testing a new flu vaccine. The experiment is to inject a patient with the vaccine and observe the occurrence of side effects. Assume that we perform this experiment 100 times and obtain the information in the table. Based on the table, if a physician injects a patient with this vaccine, what is the probability that the patient will develop severe side effects?

Side Effects	None	Mild	Severe
# of Times	72	25	3

Example: Using Empirical Information to Assign Probabilities (cont)

Solution

$$P(E) = \frac{\text{the number of times } E \text{ occurs}}{\text{the number of times the experiment is performed}}$$

$$= \frac{3}{100} = 0.03$$

Side Effects	None	Mild	Severe
# of Times	72	25	3

Example: Investigating Marital Data

The table summarizes the marital status of men and women in the United States between the ages of 18 and 34 in a recent year. All numbers represent thousands. If we randomly pick a female, what is the probability that she is married but not separated?

	Now Married (except separated)	Widowed	Divorced or Separated	Never Married
Males	9,740	36	1,621	24,655
Females	12,279	104	2,328	20,449

Example: Investigating Marital Data (cont)

Solution

We are only interested in females, so we consider our sample space to be the

12,279 + 104 + 2,328 + 20,449 = 35,160 females

The event, call it *F*, is the set of 12,279 thousand females who are married but not separated.

	Now Married (except separated)	Widowed	Divorced or Separated	Never Married
Males	9,740	36	1,621	24,655
Females	12,279	104	2,328	20,449

Example: Investigating Marital Data (cont)

Thus the probability that we would select a married female who is not separated is

$$P(F) = \frac{n(F)}{n(S)}$$

$$= \frac{12{,}279}{35160}$$

$$\approx 0.349 \ or \ 0.35 \ or \ 35\%$$

	Now Married (except separated)	Widowed	Divorced or Separated	Never Married
Males	9,740	36	1,621	24,655
Females	12,279	104	2,328	20,449

Counting and Probability

Probabilities may be based on *empirical information*, for example, the result of experimental data.

Probabilities may be based on *theoretical information*.

Calculating Probability When Outcomes Are Equally Likely

If E is an event in a sample space S with all *equally likely outcomes,* then the probability of E is given by the formula:

$$P(E)=\frac{n(E)}{n(S)}.$$

Example: Using Counting to Calculate Probabilities

We flip three fair coins. What is the probability of each outcome in this sample space?

Solution

Eight equally likely outcomes are shown below. Each has a probability of 1/8.

Example: Computing Probability of Events

a) What is the probability in a family with three children that two of the children are girls?

Solution

We denote the event that two of the children are girls by the set
G = {ggb, gbg, bgg}.

$$P(G) = \frac{n(G)}{n(S)} = \frac{3}{8}$$

Example: Computing Probability of Events (cont)

b) What is the probability that a total of four shows when we roll two fair dice?

Solution

The sample space for rolling two dice is a set of 36 ordered pairs of numbers, which again we call *S*.

“roll a total of four” as the set

$F = \{(1, 3), (2, 2), (3, 1)\}$

$$P(F) = \frac{n(F)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Basic Properties of Probability

Assume that S is a sample space for some experiment and E is an event in S.

1. $0 \leq P(E) \leq 1$

2. $P(\varnothing) = 0$

3. $P(S) = 1$

Odds

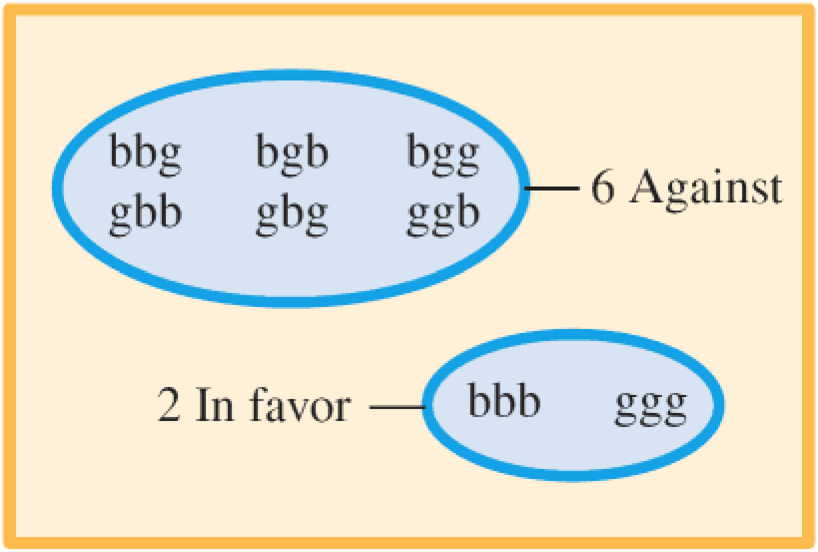
If the outcomes of a sample space are equally likely, then the **odds against an event** *E* are simply the number of outcomes that are against *E* compared with the number of outcomes in favor of *E* occurring. We write these odds as n(*E*'): *n*(*E*), where *E*' is everything that is not event *E*.

Draw a Venn diagram representing n(E) and n(E').

Odds

If a family has 3 children, what are the odds against all 3 children being of the same gender? 6:2 or 3:1

What are the odds in favor? 1:3



Example: Calculating Odds on a Roulette Wheel

A common type of roulette wheel has 38 equal-size compartments. Thirty-six of the compartments are numbered 1 to 36, with half of them colored red and the other half black. The remaining 2 compartments are green and numbered 0 and 00. A small ball is placed on the spinning wheel, and when the wheel stops, the ball rests in one of the compartments. What are the odds against the ball landing on red?

Example: Calculating Odds on a Roulette Wheel (cont)

Solution

This is an experiment with 38 equally likely outcomes. Because 18 of these are in favor of the event “the ball lands on red” and 20 are against the event, the odds against red are 20 to 18. We can write this as 20:18, which we may reduce to 10:9.

Odds

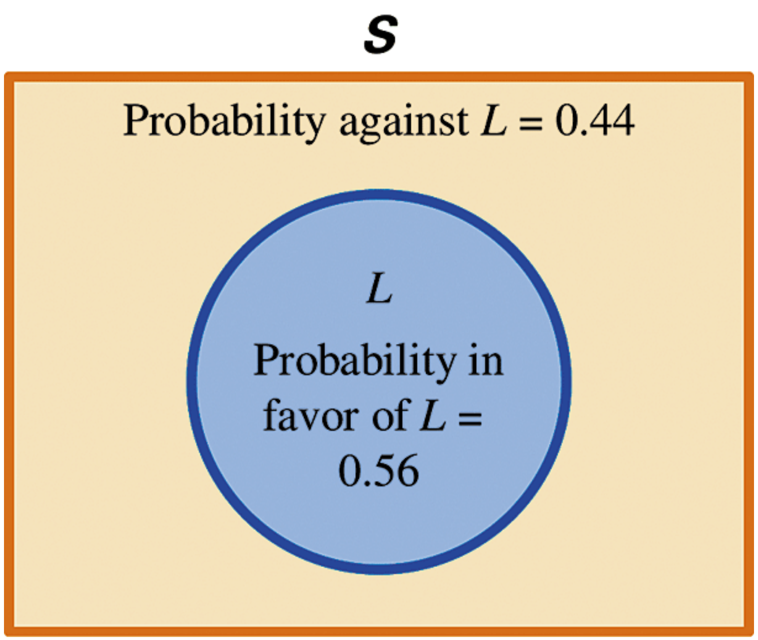
Probability Formula for Computing Odds

If *E'* is the complement of the event *E*, then the odds against *E* are

$$\frac{P(E')}{P(E)}.$$

Example: The Odds Against Surviving an Airplane Crash

The safest place to be seated in the event of an airplane crash is in an aisle seat above the wings. In this case, the probability of surviving the crash is 56%. What are the odds against you surviving?



Example: The Odds Against Surviving an Airplane Crash (cont)

Solution

$$\frac{P(L')}{P(L)} = \frac{0.44}{0.56} = \frac{0.44 \times 100}{0.56 \times 100} = \frac{44}{53} = \frac{11}{14}$$

Thus we would say that if you sit over the wings, the odds against you surviving are 11 to 14.