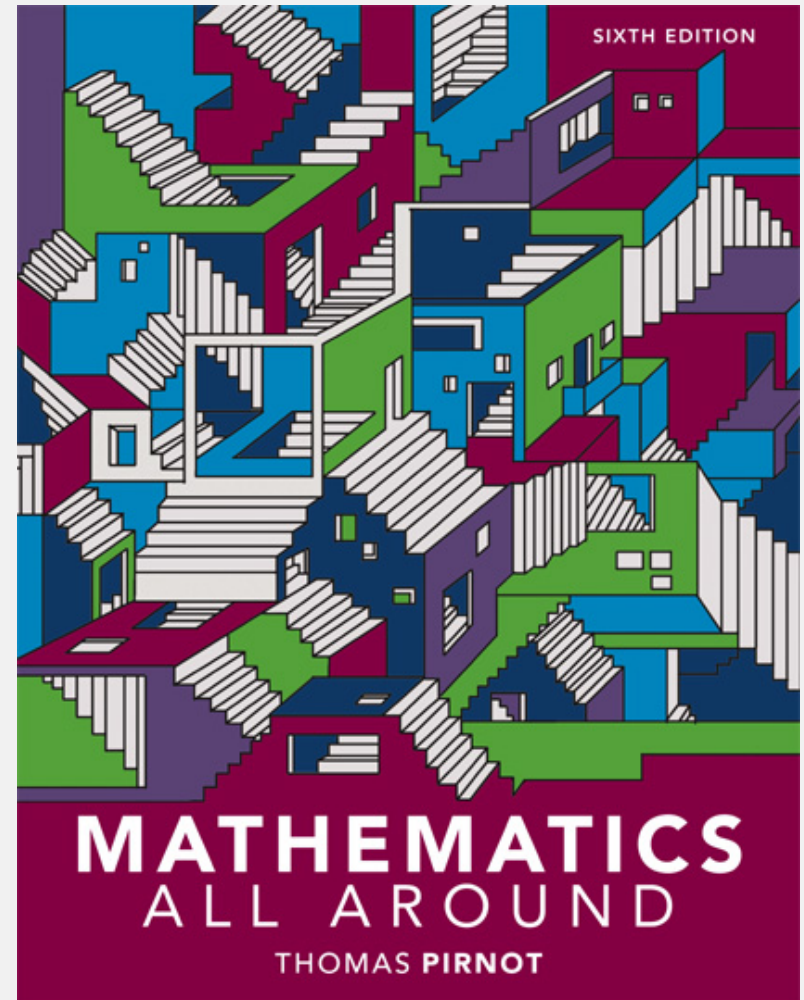


1

Strategies and Principles



1.2 Inductive and Deductive Reasoning

- Use inductive reasoning to make conjectures.
- Give examples of correct and incorrect inductive reasoning.
- Be able to distinguish between inductive and deductive reasoning.

Inductive Reasoning

Inductive reasoning is the process of drawing a general conclusion by observing a pattern in specific instances. This conclusion is called a **hypothesis** or **conjecture**.

Example: A Divisibility Test for 9

Consider the numbers a) 72, b) 491, c) 963, d) 19,856, e) 45,307, and f) 7,538,463. Verify that the numbers a), c), and f) are evenly divisible by 9, but b), d), and e) are not. Add the digits of each number. Do you see any pattern? Make a conjecture.

(Solution on the next slide.)

Example: A Divisibility Test for 9 (cont)

Solution

a) 72: $7 + 2 = 9$

b) 491: $4 + 9 + 1 = 14$

c) 963: $9 + 6 + 3 = 18$

d) 19,856: $1 + 9 + 8 + 5 + 6 = 29$

e) 45,307: $4 + 5 + 3 + 0 + 7 = 19$

f) 7,538,463: $7 + 5 + 3 + 8 + 4 + 6 + 3 = 36$

Example: A Divisibility Test for 9 (cont)

Notice that for the numbers a), c), and f) that were divisible by 9, the sum of their digits is evenly divisible by 9 and for the numbers b), d), and e) that were not divisible by 9, the sum of their digits is not divisible. Our conjecture is that in order for 9 to divide evenly into a number, 9 must divide the sum of the digits of the number.

Problem Solving



Problem Solving

Strategy: The Always Principle

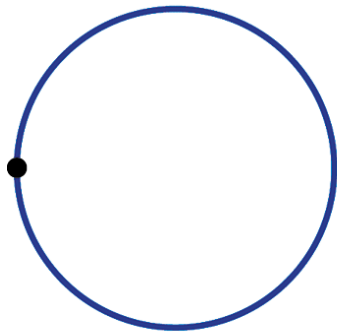
Keep in mind that in doing inductive reasoning, you are only making an educated guess. You cannot be sure that your conclusion is true. Recall our discussion of the

Always Principle and the Counterexample Principle in Section 1.1.

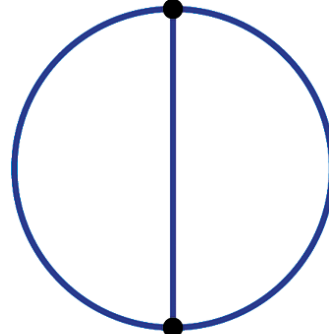
Example: False Inductive Reasoning

We want to divide a circle into regions by selecting points on its circumference and drawing line segments from each point to each other point. The figure (on the next slide) shows the greatest number of regions that we get if we have one point (no line segment is possible for this case), two, three, and four points. Use inductive reasoning to find the greatest number of regions we would get if we had six points on the edge of the circle.

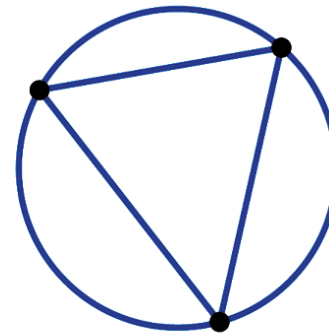
Example: False Inductive Reasoning (cont)



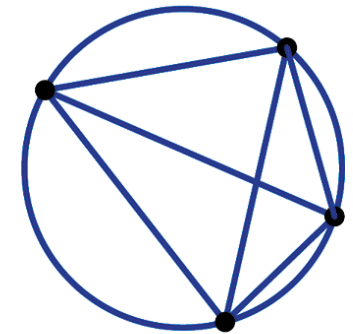
(a)
One point;
one region



(b)
Two points;
two regions



(c)
Three points;
four regions



(d)
Four points;
eight regions

Solution

It appears that each time we add another point, we double the number of regions. It is natural to conjecture that if we have five points, there would be 16 regions, and with six points, we would get 32 regions. However, this is not true. Try it for yourself by drawing a large circle and picking six points in different ways on the circle. The largest number of regions that you will find is 31, not 32.

Deductive Reasoning

In **deductive reasoning**, we use accepted facts and general principles to arrive at a specific conclusion.

Examples of deductive reasoning:

- Mathematical proofs

- Step-by-step mathematical solutions

- Using scientific laws to make predictions

Example: Using Deductive Reasoning to Solve a Puzzle

Four students, Alex, Carmella, Noah, and Winnie, participate in different college activities (debate team, basketball, orchestra, or theater). Use the following clues to determine the activity of each student.

1. Winnie lives in the same apartment complex as the musician and theater participant.
2. The musician and Noah were friends in high school.

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

3. Carmella has a heavier course load than the basketball player, but fewer credits than the theater participant.
4. Noah, who has the fewest credits, is not on the debate team.

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

Solution

To organize our thinking, we will list all possibilities in a table. From clue 1, we see that Winnie is not the musician and does not participate in theater, which allows us to place two X's in the table to show that we have eliminated two possibilities.

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

Solution

To organize our thinking, we will list all possibilities in a table. From clue 1, we see that Winnie is not the musician and does not participate in theater, which allows us to place two X's in the table to show that we have eliminated two possibilities. (see next slide)

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

	A	C	N	W
Debate				
Basketball				
Orchestra				X-Clue 1
Theater				X-Clue 1

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

Clue 2 tells us that Noah does not play in the orchestra; from clue 3 we deduce that Carmella is not the basketball player and does not participate in theater. We can place three more X's in the table.

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

	A	C	N	W
Debate				
Basketball		X-Clue 3		
Orchestra			X-Clue 2	X-Clue 1
Theater		X-Clue 3		X-Clue 1





Example: Using Deductive Reasoning to Solve a Puzzle (cont)

Clue 4 tells us that Noah is not on the debate team, but also because Noah has the fewest credits, by clue 3 he cannot be participating in theater. Our table now looks like this.

	A	C	N	W
Debate			X-Clue 4	
Basketball		X-Clue 3		
Orchestra			X-Clue 2	X-Clue 1
Theater		X-Clue 3	X-Clue 4	X-Clue 1

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

This forces Noah to be the basketball player, which means neither Winnie nor Alex plays basketball. Now it is clear that Winnie is on the debate team, therefore we can rule out Alex and Carmella as debaters. This forces us to conclude that Alex is involved with theater.

	A	C	N	W
Debate	X	X	X-Clue 4	
Basketball	X-Clue 4	X-Clue 3		X-Clue 4
Orchestra	X		X-Clue 2	X-Clue 1
Theater		X-Clue 3	X-Clue 4	X-Clue 1

Example: Using Deductive Reasoning to Solve a Puzzle (cont)

So, Alex is participating in theater, Carmella is in the orchestra, Noah is the basketball player, and Winnie is doing debate.

Outside of Class

- ❖ Study *Example 6 – Done in class (pg. 25)*
- ❖ Turn in *Exercise #39*