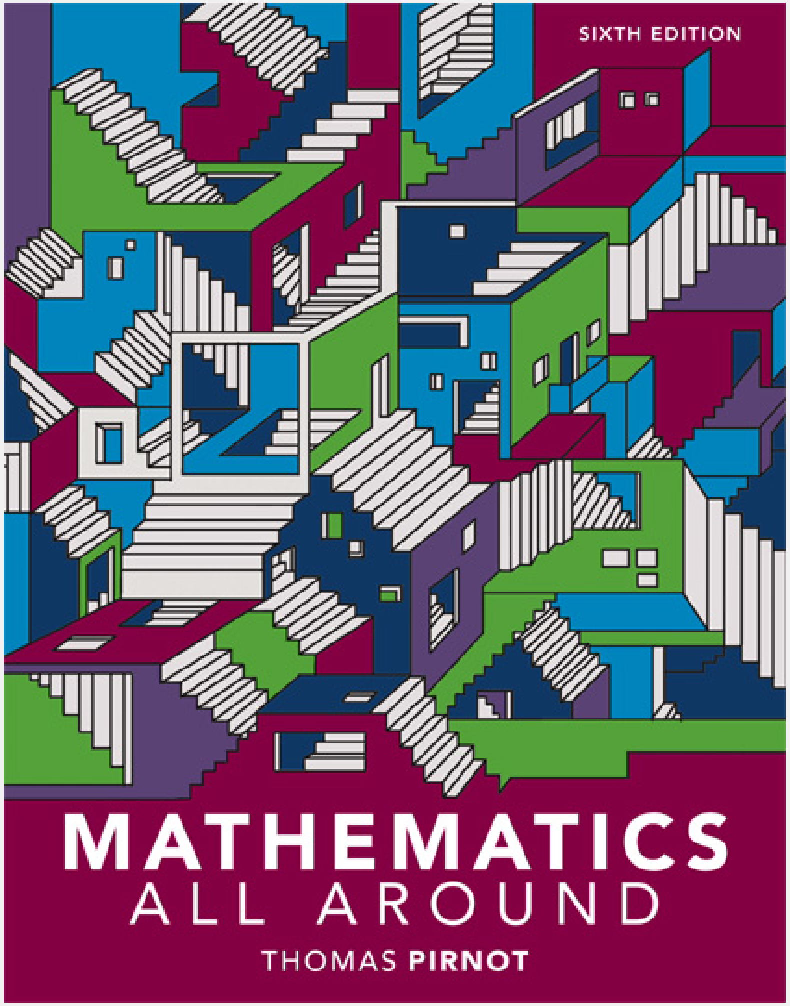
1

Strategies and Principles



Goals of this Course

- Your college education should prepare you not just for today, or even tomorrow, but for a long, productive life full of changes and problems to be solved.

- Improve you ability to *THINK* creatively and critically by analyzing and solving problems.

1.1 Problem Solving

- Use Polya's method to solve problems.
- State and apply fundamental problem-solving strategies.
- Apply basic mathematical principles to problem solving.
- Use the Three-Way Principle to learn mathematical ideas.

George Polya's Problem-Solving Method

Step 1: Understand the problem.

Step 2: Devise a plan.

Step 3: Carry out your plan.

Step 4: Check your answer.

Example of a Problem

How many quarters – placed one on top of the other – would it take to reach the top of the Empire State Building in New York City?

Problem-Solving Strategies

Problem Solving

Strategy: Draw Pictures

Problems usually contain several conditions that must be satisfied. You will find it useful to draw pictures to understand these conditions before trying to solve the problem.

Problem-Solving Strategies

Problem Solving

Strategy: Choose Good Names for Unknowns

It is a good practice to name the objects in a problem so you can remember their meaning easily.

Example: Visualizing a Condition in a Word Problem and Naming

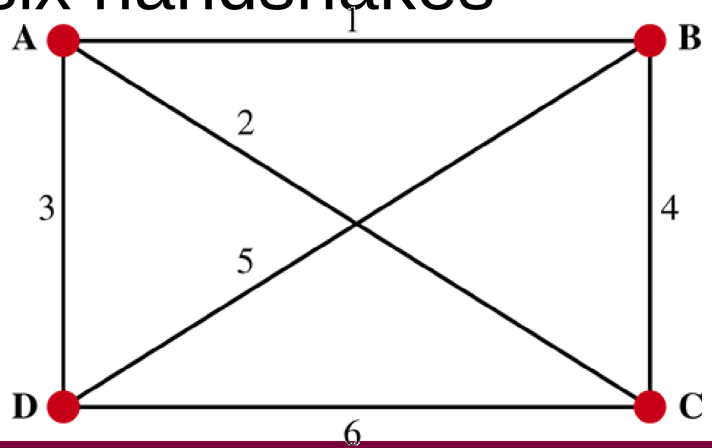
Four campers, Adliya, Benjamin, Christine, and Dari, have just arrived at the Seeds of Peace Camp in Maine for an orientation session. Each will shake hands with all of the others. Draw a picture to illustrate this situation, and determine the number of handshakes.

Example: Visualizing a Condition in a Word Problem (cont)

SOLUTION

We will use points labeled A, B, C, and D, respectively, to represent the people, and join these points with lines representing the handshakes, as shown.

If we represent the handshake between A and B by AB, then we see that there are six handshakes—namely, AB, AC, AD, BC, BD, and CD.



Example 2: Combining the Naming Strategy and the Drawing Strategy

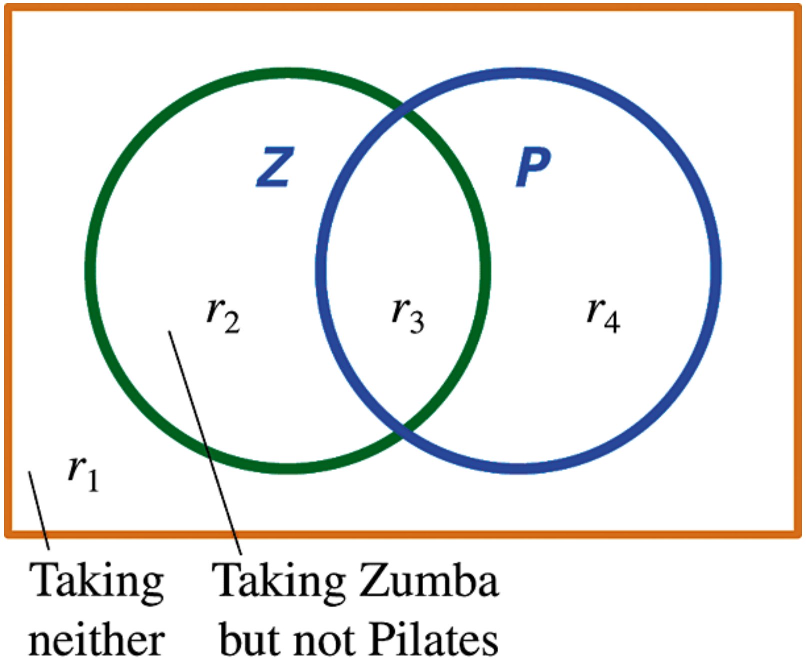
Assume that one group of students who are interested in physical fitness is taking Zumba classes and another is taking Pilates. Choose good names for these groups and represent this situation with a diagram.

Example 2: Combining the Naming Strategy and the Drawing Strategy (cont)

Solution: In the figure, the region labeled Z represents the students taking Zumba and the region labeled *P* represents the students taking Pilates.

As you can see in the Figure, the region marked r_2 indicates students who are taking Zumba but not Pilates.

Region r_1 represents students who are taking neither.



Problem-Solving Strategies

Problem Solving

Strategy: Be Systematic

If you approach a situation in an organized, systematic way, frequently you will gain insight into the problem.

Example: Systematically Listing Options

Nico is considering which optional features to include with his new smartphone. He has narrowed it down to three choices: quick-charging battery, multiple windows, and a high-resolution camera lens. Depending on price, he will decide how many of these options he can afford. In how many ways can he make his decision?

Example: Systematically Listing Options (cont)

Solution: In Nico's decision, we see that there are four cases. Choose none *or* one *or* two *or* all three of the options.

	Battery	Windows	Camera
choose none	No	No	No
choose one	Yes	No	No
	No	Yes	No
	?	?	?
choose two	Yes	Yes	No
	Yes	No	Yes
	?	?	?
choose three	Yes	Yes	Yes

Problem-Solving Strategies

Problem Solving

Strategy: Look for Patterns

If you can recognize a pattern in a situation you are studying, you can often use it to answer questions about that situation.

Problem Solving

Strategy: Try a Simpler Version of the Problem

You can begin to understand a complex problem by solving some scaled-down versions of the problem. Once you recognize a pattern in the way you are solving the simpler problems, you can carry over this insight to attack the full-blown problem.

Example 3

Find the last digit in the number 2^{100}

(that is 2 to the 100^{th} power!)

- *Turn in as Assignment #1 for next class*
- *You may work as a team with another classmate.*

Problem-Solving Strategies

Problem Solving

Strategy: Guessing Is OK

One of the difficulties in solving word problems is that you can be afraid to say something that may be wrong and consequently sit staring at a problem, writing nothing until you have the full-blown solution. Making guesses, even incorrect guesses, is not a bad way to begin. It may give you some understanding of the problem. Once you make a guess, evaluate it to see how close you are to meeting all the conditions of the problem.

Problem-Solving Strategies

Problem Solving

Strategy: Relate a New Problem to an Older One

An effective technique in solving a new problem is to try to connect it with a problem you have solved earlier. It is sometimes possible to rewrite a condition so that the problem becomes exactly like one you have seen before.

Some Mathematical Principles

Problem Solving

Strategy: The Always Principle

When we say a statement is true in mathematics, we are saying that the statement is true *100% of the time*. One of the great strengths of mathematics is that we do not deal with statements that are "sometimes true" or "usually true."

Some Mathematical Principles

Problem Solving

Strategy: The Counterexample Principle

An example that shows that a mathematical statement fails to be true is called a **counterexample.** Keep in mind that if you want to use a mathematical property and someone can find a counterexample, then the property you are trying to use is not allowable. A hundred examples in which a statement is true do not prove it to be *always* true, yet a single example in which a statement fails makes it a false statement. Be careful to understand that when we say a statement is false, we are not saying that it is always false. We are only saying that the statement is *not always true*. That is, we can find at least one instance in which it is false.

Some Mathematical Principles

Problem Solving

Strategy: The Order Principle

When you read mathematical notation, pay careful attention to the *order* in which the operations must be performed. The order in which we do things in mathematics is as important as it is in everyday life. When you get dressed in the morning, it makes a difference whether you first put on your socks and then your shoes, or first put on your shoes and then your socks. Although the difference may not seem as dramatic, reversing the order of mathematical operations can also give unacceptable results. Note that we are not saying that it is *always* wrong to reverse the order of mathematical operations; we are saying that if you reverse the order of operations, you *may* accidentally change the meaning of your calculations.

Some Mathematical Principles

Problem Solving

Strategy: The Splitting-Hairs Principle

You should "split hairs" when reading mathematical terminology. If two terms are similar but sound slightly different, they usually do not mean exactly the same thing. In everyday English, we may use the words *equal* and *equivalent* interchangeably; however, in mathematics they do not mean the same thing. The same is true for notation. When you encounter different-looking notation or terminology, work hard to get a clear idea of exactly what the difference is. Representing your ideas precisely is part of good problem solving.

Some Mathematical Principles

Problem Solving

Strategy: The Analogies Principle

Much of the formal terminology that we use in mathematics sounds like words that we use in everyday life. This is not a coincidence. Whenever you can associate ideas from real life with mathematical concepts, you will better understand the meaning behind the mathematics you are learning.

Some Mathematical Principles

Problem Solving

Strategy: The Three-Way Principle

We conclude this section with a method for approaching mathematical concepts that we illustrate in Figure 1.6.

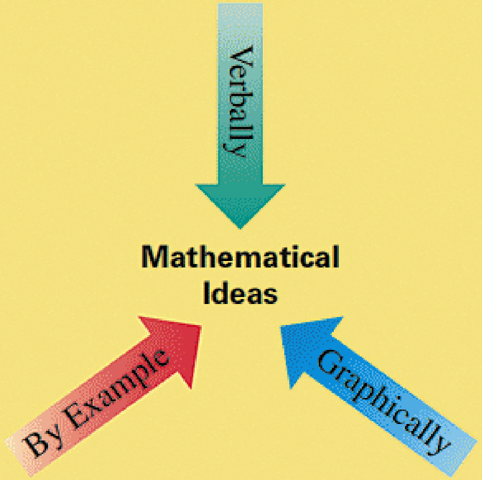


FIGURE 1.6 The Three-Way Principle.

Whether you are learning a new concept or trying to gain insight into a problem, it is helpful to use the ideas we have discussed in this chapter to approach mathematical situations in three ways.

- *Verbally*—Make analogies. State the problem in your own words. Compare it with situations you have seen in other areas of mathematics.
- *Graphically*—Draw a graph. Draw a diagram.
- *By example*—Make numerical or other kinds of examples to illustrate the situation.

Not every one of these three approaches fits every situation. However, if you get in the habit of using a verbal-graphical-example approach to doing mathematics, you will find that mathematics is more meaningful and less dependent on rote memorization. If you practice approaching mathematics using the strategies and principles that we have discussed, you will find eventually that you are more comfortable and more successful in your mathematical studies.

Outside of Class

- Study *Example #8 (pg. 10)*
- Do *Exercises #65 - #68*