University of North Georgia
Quantitative Skills and Reasoning - Fall 2019
Exam #3 - M. Goodroe

Name: _Key_

Directions: Answer each question completely on the exam. Seventy-five percent of the total points for a question will be awarded to your explanation and twenty-five percent to the correct answer. Each question is worth four points.

Solve the problem.

1) John owns a hotdog stand. His profit is represented by the equation $P = -x^2 + 10x + 31$, with P being profits and x the number of hotdogs. What is the most he can earn?

$$x = -\frac{b}{2a} = \frac{-10}{-2} = 5 \qquad \text{Vertex}: (5, \$56)$$

$$-(5)^2 + 10(5) + 31$$
$$-25 + 50 + 31 \qquad \$56 \text{ max profit}$$
$$25 + 31$$
$$56$$

2) Assume that the mean length of a human pregnancy is 268 days with a standard deviation of 9 days. What percentage of human pregnancies do we expect to last no more than 260 days?

$$\mu = 268 \text{ days} \qquad z = \frac{260 - 268}{9} = -\frac{8}{9} = -0.89$$
$$\sigma = 9 \text{ days}$$

$$\boxed{18.67\%}$$

3) A projectile is thrown upward so that its distance, in feet, above the ground after t seconds is $h = -16t^2 + 416t$. What is its maximum height?

$$x = -\frac{416}{-32} = \frac{-416}{-32} = 13 \qquad \text{Vertex}: (13, 2704)$$

$$-16(13)^2 + 416(13) \Big| = 2704 \qquad \boxed{2,704 \text{ ft max height}}$$
$$-16(169) + 5408$$
$$-2704 + 5408$$

4) If two fair dice are rolled, what is the probability that a total showing is either even or less than nine?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad \overset{A}{} \qquad \overset{B}{}$$

$$= \frac{18}{36} + \frac{26}{36} - \frac{14}{36}$$

$$= \frac{30}{36}$$

$$= \boxed{\frac{5}{6}}$$

1

Solve the problem. Assume that the given distribution is normal.

5) Due to random variations in the operation of an automatic hot chocolate machine, not every cup is filled with the same amount of hot chocolate. Assume that the mean amount dispensed is 12 ounces with a standard deviation of 0.7 ounces. What percentage of the cups should have more than 12.7 ounces of hot chocolate?

$\mu = 12$ oz

$\sigma = 0.7$ oz

50% | 34%

12   12.7 oz
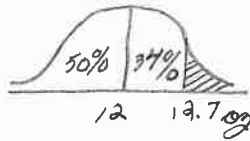
$100 - (50 + 34) = \boxed{16\%}$

or $z = \dfrac{12.7 - 12}{0.7} = \dfrac{0.7}{0.7} = 1$

$\downarrow$

0.8413 and $1 - 0.8413 = 0.1587$

$\boxed{15.87\% \text{ or } 16\%}$

Determine whether the argument is valid or invalid.

6) If the pond contains algae, then the pond does not contain trout.
If the pond does not contain algae and does not contain trout, then the pond does not contain frogs.
The pond contains algae.
Therefore, the pond contains frogs.

$p \rightarrow \sim q$

$(\sim p \wedge \sim q) \rightarrow \sim r$

p

$\therefore r$

$\boxed{\text{Invalid}}$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$(p \rightarrow \sim q)$	$(\sim p \wedge \sim q) \rightarrow \sim r$	
T	T	T	F	F	F	F	F	
T	T	F	F	F	T	F	F	
T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	T	F	T
F							F	
F							F	
F							F	
F							F	

7) As a car begins to accelerate, the gas mileage is poor. As the speed increases, the gas mileage continues to increase gas mileage increases for a while, then as the speed increases further, the mileage begins to decrease. The table of data below illustrates this.

$a = m$ b

$y = \boxed{0.06} x + \boxed{25.8}$

$r = 0.3974$

Speed in Miles per Hour	Mileage in Miles per Gallon
30	25
40	30
50	31
60	30
70	28

Find the line of best fit for the data and the Correlation Coefficient (r-value). Interpret the "rate of change" and Correlation Coefficient in context of the situation.

Slope $= a = m = 0.06 : \dfrac{6}{100} = \dfrac{3}{50}$ as speed increases by 50 mph 3 mpg of gas is used

y-int. $= 25.8$ starting gas mileage

r-value: 0.397 says there is a weak correlation between speed (mph) and mileage (mpg).

Find the indicated probability.

8) You roll two fair dice. Let E be the event that an even total shows on the dice. Let F be the event that a two shows on at least one of the dice. Find P(F) and P(F|E).

For five extra points, determine if the events E and F are independent or dependent, that is does P(F|E) = P(F) or not.

$$P(F) = \boxed{\dfrac{11}{36}}$$
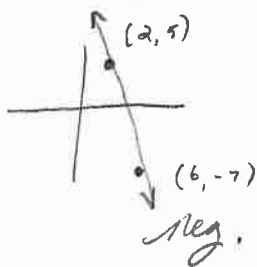
$$P(E) = \dfrac{18}{36} = \dfrac{1}{2}$$

$$P(F|E) = \dfrac{P(F \cap E)}{P(E)} = \dfrac{\frac{5}{36}}{\frac{18}{36}} = \dfrac{5}{36} \cdot \dfrac{36}{18} = \boxed{\dfrac{5}{18}}$$

Extra Credit: If $P(F|E) = P(F)$, the events F and E are independent

$$\dfrac{5}{18} \neq \dfrac{11}{36}, \text{ therefore } F \text{ and } E \text{ are dependent.}$$

Find a linear equation whose graph passes through the given points. Write the equation in slope–intercept form.

9) (2, 5) and (6, –7)

$$m = \dfrac{(-7) - (5)}{(6) - (2)} = \dfrac{-12}{4} = \dfrac{-3}{1}$$

$$y = mx + b$$
$$5 = -3(2) + b$$
$$5 = -6 + b$$
$$11 = b$$

$$\boxed{y = -3x + 11}$$

neg.

Interpret the meaning of the slope and the y–intercept of the equation in terms of the conditions stated in the problem.

10) The cost, in dollars, of retaining the services of a computer repairman in Anchorville is given by y = 42x + 32, where x is the number of hours worked.

Slope: $\dfrac{42}{1}$ per one hour worked the cost is $42.00 or rate per hour worked.

y-int.: (0, 32) it costs $32.00 for repairman's initial services.

Use the complement formula to answer the question.

11) If two fair dice are rolled, what is the probability that a total showing is more than two?

$$\boxed{\dfrac{35}{36}}$$

$$1 - P(E') = P(E)$$
$$\dfrac{36}{36} - \dfrac{1}{36} = \dfrac{35}{36}$$

3

Using the below Quadratic equation, find the following:

a.) How the Graph opens: *down*; $a = -1$

b.) y–Intercept: $(0, -12)$

c.) x–Intercepts: *not real*

d.) Vertex: $(2, -8)$

e.) The Axis of Symetry: $x = 2$

12) $y = -x^2 + 4x - 12$

$$x = \frac{-(4) \pm \sqrt{4^2 - 4(-1)(-12)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{16 - 48}}{-2}$$

$$= \frac{-4 \pm \sqrt{-32}}{-2} \longleftarrow \text{no Real Solutions}$$

$$= \frac{-4 \pm 4i\sqrt{2}}{2}$$

$$= -2 \pm 2i\sqrt{2} \quad \text{not Real}$$

Vertex: $(2, -8)$

$$x = -\frac{4}{2(-1)} = \frac{-4}{-2} = 2$$

$$-(2)^2 + 4(2) - 12$$

$$-4 + 8 - 12$$

$$4 - 12$$

$$-8$$

4

1) $56

2) 18.7%

3) 2704 ft

4) $\dfrac{5}{6}$

5) 16%

6) Invalid

7) $y = 0.06x + 25.8$

8) $P(F) = \dfrac{11}{36}$, $P(F|E) = \dfrac{5}{18}$

9) $y = -3x + 11$

10) 42 signifies the hourly rate, and 32 signifies the overhead charge.

11) $\dfrac{35}{36}$

12) $(2, -8)$, down