University of North Georgia
Department of Mathematics

Instructor: Berhanu Kidane
Course: College Algebra Math 1111
Text Book: For this course we use the free e-book by Stitz and Zeager with link:
Other online resources:
   e-book: http://msenux.redwoods.edu/IntAlgText/

For more free supportive educational resources consult the syllabus
Chapter 2
Quadratic Functions (page 151)

2.3 Quadratic Functions and Models (page 188)

Objectives: By the end of this section students should be able to

- Identify quadratic functions
- Identify the standard form of quadratic functions
- Find maximum and minimum points and values of quadratic functions
- Find a vertex or turning point of a quadratic function
- Graph quadratic functions
- Solve applied problems that use quadratic functions

Quadratic Functions

Definition:
A quadratic function is a function of the form \( f(x) = ax^2 + bx + c \), \( a \neq 0 \) where \( a \), \( b \), and \( c \) are called coefficients. A quadratic function is a polynomial function of degree 2.

Example 1:

a) \( f(x) = x^2 \)

b) \( f(x) = 2x^2 - 3x + 12 \)

Standard form of a quadratic function
A quadratic function \( f(x) = ax^2 + bx + c \), \( a \neq 0 \) using completing the form can be expressed in the Standard Form \( f(x) = a(x - h)^2 + k \)

Example 2: The following are in standard forms

a) \( f(x) = x^2 \)

b) \( f(x) = -2(x - 1)^2 + 2 \)

The Graph of a Quadratic Function

The graph of \( f(x) = ax^2 + bx + c \), \( a \neq 0 \) is called a parabola which either opens up or opens down. The parabola opens up if \( a > 0 \) and opens down if \( a < 0 \)

i. Standard form: \( f(x) = a(x - h)^2 + k \)
   - \((h, k)\) is called vertex, which is the highest or lowest point,
     - \((h, k)\) is the highest point if \( a < 0 \); and \( f(h) = k \) is the maximum value
     - Domain = \((-\infty, \infty)\) and Range = \((-\infty, k]\)
     - \((h, k)\) is the lowest point if \( a > 0 \); and \( f(h) = k \) is the minimum value
     - Domain = \((-\infty, \infty)\) and Range = \([k, \infty))
   - The line \( x = h \) is called the axis of symmetry
ii. General form: \( f(x) = ax^2 + bx + c. \quad a \neq 0 \)

- Vertex is \( \left( \frac{-b}{2a}, \ f\left(\frac{-b}{2a}\right) \right) = \left( \frac{-b}{2a}, \ \frac{4ac - b^2}{4a} \right) \); Vertex is also called Turning Point
  - If \( a > 0 \), then the minimum value is \( f\left(\frac{-b}{2a}\right) \)
  - Domain \( = (-\infty, \infty) \) and Range \( \left[ f\left(\frac{-b}{2a}\right), \ \infty \right) \)
  - If \( a < 0 \), then the maximum value is \( f\left(\frac{-b}{2a}\right) \)
  - Domain \( = (-\infty, \infty) \) and Range \( \left( -\infty, f\left(\frac{-b}{2a}\right) \right] \)

- The axis of symmetry is the line \( x = \frac{-b}{2a} \)

- The function \( f(x) = ax^2 + bx + c. \quad a \neq 0 \) has the standard form
  \[
  f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}
  \]

**Example 3:** For the graphs of the parabolas below find the vertex, the maximum or the minimum point, the maximum or the minimum value, and the axis of symmetry

**Examples:**

- Example 2.3.1 Page 188 Reading
- Example 2.3.2 Page 190 Reading

**Example 4:** Write the following in the standard form

\[
\begin{align*}
a) \quad f(x) &= 2x^2 - 12x + 23 \\
b) \quad f(x) &= 3x^2 - 6x + 1
\end{align*}
\]

**OER 1:** West Texas A&M University Tutorial 34: Graphs of Quadratic Functions

**OER 2:** Notes and Exercises

a) Chapter 5: Quadratic Functions
   a. Section 1: The Parabola
      i. Exercises with Answers
   b. Section 2: Vertex Form
      i. Exercises with Answers
   c. Section 5: Motion (#29 – 40 Odd numbered problems)
      i. Exercises with Answers
   d. Section 6: Optimization (#1 – 38 odd numbers)
      i. Exercises with Answers
3 ways to find a vertex or turning point of a quadratic function:

1) If in the standard form, \( f(x) = a(x - h)^2 + k \), then vertex = \((h, k)\)

2) If not in the standard form, then use the formula \( \left( \frac{-b}{2a}, f \left( \frac{-b}{2a} \right) \right) = \left( \frac{-b}{2a}, \frac{4ac - b^2}{4a} \right) \).

3) Completing the square if not in standard form.

Example 5: Find the vertex, maximum and minimum values and the range of each.

a) \( f(x) = x^2 + 10x + 23 \)  
b) \( f(x) = (x - 2)^2 + 3 \)

Example 6: Find the vertex, axis of symmetry, the minimum or maximum point, intervals where increasing and decreasing, and graph each.

a) \( f(x) = (x + 3)^2 - 1 \)  
b) \( f(x) = -2x^2 + 8x - 6 \)

Example 2.3.3: Page 196 Reading

Example 2.3.4: Page 197 Reading

Example 7: Suppose \( C(x) = x^2 - 10x + 27 \) represents the costs, in hundreds, to produce \( x \) thousand pens. How many pens should be produced to minimize the cost? What is this minimum cost?

Example 8: A stone is thrown directly upward from a height of 30 ft with an initial velocity of 60 ft/sec. The height of the stone \( t \) seconds after it has been thrown is given by the function \( S(t) = -16t^2 + 60t + 30 \). Determine the time at which the stone reaches its maximum height and find the maximum height.

Example 9: Skippy wishes to plant a vegetable garden along one side of his house. In his garage, he found 32 linear feet of fencing. Since one side of the garden will border the house, Skippy doesn’t need fencing along that side. What are the dimensions of the garden which will maximize the area of the garden? What is the maximum area of the garden?

Example 10: What is the largest rectangular area one can enclose with 14 inches of string?

Example 11: The height of an object dropped from the roof of an eight story building is modeled by \( h(t) = -16t^2 + 64, \ 0 \leq t \leq 2 \). Here, \( h \) is the height of the object off the ground, in feet, \( t \) seconds after the object is dropped. How long it takes before the object hits the ground?

Example 11: Carl’s friend Jason participates in the Highland Games. In one event, the hammer throw, the height \( h \) in feet of the hammer above the ground \( t \) seconds after Jason lets it go is modeled by \( h(t) = -16t^2 + 22.08t + 6 \). What is the hammer’s maximum height? What is the hammer’s total time in the air? Round your answers to two decimal places.

Practice Problems from the Text
Page 200, Exercise 2.3.1: 1 – 34 Odd numbers
3.1 Polynomial Functions and Their Graphs

Objectives: By the end of this section students should be able to:

- Identify polynomial Functions
- Graph basic polynomial functions
- Identify end behavior and leading term
- Use zeros in graphing polynomials
- Find local maximum and local minimum

Definition of a Polynomial Function

A polynomial function of degree \( n \) is a function of the form

\[ p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0, \]

where \( n \) is a non-negative integer and \( a_n \neq 0 \).

- The numbers \( a_n, a_{n-1}, \ldots, a_1, a_0 \) are called the coefficients of the polynomial.
- The number \( a_0 \) is called the constant coefficient or constant term.
- The number \( a_n \), the coefficient of the highest power, is the leading coefficient.
- The term \( a_n x^n \) is the leading term.
- \( p(x) = a_0 \) is a polynomial of degree 0.
- If \( p(x) = 0 \), we say \( p \) has no degree.

Example 3.1.1 Page 235 reading: Which of the following functions are polynomials?

Example 1: Which of the following is a polynomial? If an expression is a polynomial, name its degree, and tell the variable that the polynomial is in.

- a) \( f(x) = x^3 - 2x^2 - 3x - 4 \)
- b) \( p(y) = 3y^2 + 2y + 1 \)
- c) \( h(x) = x^3 + 2\sqrt{x} + 1 \)
- d) \( f(x) = 3\sqrt{x^2} \)
- e) \( f(z) = z^{-1} + 2 \)

Example 2: \( p(x) = -2x^5 + x^4 + 3x^3 - 4x^2 - 12x + 6 \)

- a) Degree
- b) Leading term
- c) Leading coefficient
- d) Constant term

Example 3.1.2 Page 237 Reading

Example 3: Give examples of polynomial function of

- a) degree 0
- b) degree 1
- c) degree 2
- d) degree 33
Example 3.1.3 Page 338: An open box will be made by cutting out congruent squares from each corner of a 3ft by 8ft piece of cardboard and then folding up the sides. Let x denote the length of the side of the square which is removed from each corner.

a) Find the volume V of the box as a function of x. Include appropriate applied domain
b) Using graphing calculator graph \( y = V(x) \) on the domain from 1) and approximate the maximum volume to two decimal places. What is the maximum volume?

Graphs of Basic Polynomials

The simplest polynomial functions are the monomials \( p(x) = ax^n, a \neq 0 \)

Example 4: Using graphing utilities sketch the graph of \( y = ax^n \), for \( n = 1, 2, 3, 4, 5 \) and so on and deduce some general properties of the graphs of \( y = ax^n \)

Recall: The coefficient \( a \) in \( y = ax^n \), for \( a > 0 \) either stretches or shrinks the graph vertically. If \( a < 0 \) the graph of \( y = ax^n \) is a reflection of \( y = |a|x^n \) across the x-axis. Therefore, we consider graphs of \( y = ax^n \) for the cases where \( a = \pm 1 \). Two cases i) \( n \) – odd and ii) \( n \) – even

i) \( n \) – odd; \( n = 1, 3, 5, 7 \) ...

a) \( a > 0 \) End behaviour: DOWN, UP
b) \( a < 0 \) End behaviour: UP, DOWN
ii) \( n \ - \ even \ ; \ n = 2, 4, 6, 8 \ldots \)

\[ y = x^n, \ n = 2, 4, 6 \]

\[ y = -x^n, \ n = 2, 4, 6 \]

Let \( P \) be a polynomial function of degree \( n \).

1. The domain of \( P \) (any polynomial function) is the set of all real numbers.
2. \( P \) is continuous for all real numbers, so there are no breaks, holes, or jumps in the graph.
3. The graph of \( P \) is a smooth curve with rounded corners and no sharp corners or cusps.
4. The graph of \( P \) has at most \( n \) x-intercepts or \( n \)-zeroes.
5. The graph of \( P \) has at most \( n - 1 \) turning points.
6. The graph of \( P \) has four types of end behaviours.

**Example 5:** Sketch the graph of the following functions using graphing utilities and find their end behavior, number of zeros and turning points.

a) \( f(x) = x^3 - 2x^2 - 3x + 2 \)

b) \( f(x) = -x^3 - 2x^2 + 3x + 2 \)

c) \( f(x) = -x^4 + 4x^2 - 1 \)

d) \( f(x) = x^4 - 4x^2 + 1 \)

e) \( P(x) = x^2(x + 2)(x + 1)(x - 1)(x - 2) \)
Example 6: Each of the following graphs cannot be a graph of a polynomial function; Why?

Let \( p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \) be a polynomial function of degree \( n \).

Table 1 shows End Behaviors for the graphs of polynomial functions of Degree \( n \) and Leading Coefficient \( a_n \).

<table>
<thead>
<tr>
<th>Leading Coefficient / Degree</th>
<th>End Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_n &gt; 0 ) \n ( n ) even</td>
<td>both ends up (up, up)</td>
</tr>
<tr>
<td>( a_n &lt; 0 ) \n ( n ) even</td>
<td>both ends down (down, down)</td>
</tr>
<tr>
<td>( a_n &gt; 0 ) \n ( n ) odd</td>
<td>left down, right up (down, up)</td>
</tr>
<tr>
<td>( a_n &lt; 0 ) \n ( n ) odd</td>
<td>left up, right down (up, down)</td>
</tr>
</tbody>
</table>

Table 1
Example 7: Classifying Polynomials by Their Graphs

i. Even Degree

\[ y = x^4 - 2x^2 \]
\[ a_n > 0, \text{ opens up or up, up} \]

\[ y = -\frac{1}{4}x^4 + 2x^2 - \frac{1}{2}x - 1 \]
\[ a_n < 0, \text{ opens down or down, down} \]

ii. Odd Degree

\[ y = 2x^3 - 2x + \frac{1}{8} \]
\[ a_n > 0; \text{ Left Down, Right UP} \]

\[ y = -x^3 + 4x^2 - 2x - 2 \]
\[ a_n < 0; \text{ Left UP, Right Down} \]
Example 8: Determine the leading term, the leading coefficient, the degree of the polynomial, and the end behavior of the graph.

a) \( f(x) = 2x^3 + 3x^2 - 5x + 4 \)

Ans. 1) leading term = \( 2x^3 \)

2) Leading coeff. = \( 2 \)

3) Degree = \( 3 \)

4) End Behavior down, up

c) \( f(x) = -x^5 + 3x^3 + 7 \)

Example: Name the degree, the leading coefficient, and the constant term of

\( h(x) = (5x + 1)(3x - 1)(2x + 5)^3 \)

Solution:

If we were to multiply out, then the degree of the product would be the sum of the degrees of each factor, thus the degree of \( h(x) = 1 + 1 + 3 = 5 \).

Note: \( h(x) = (5x + 1)(3x - 1)(2x + 5)^3 = (5x + 1)(3x - 1)(2x + 5)(2x + 5)(2x + 5) \).

The leading coefficient would be the product of all the leading coefficients: \( 5 \cdot 3 \cdot 2^3 = 15 \cdot 8 = 120 \).

And the constant term would be the product of all the constant terms: \( 1 \cdot (-1) \cdot 5^3 = -1 \cdot 125 = -125 \).

Example 9: Find the degree, the leading coefficient, and the constant term.

a) \( f(x) = 6x^3 + 7x^2 - 3x + 1 \)

b) \( f(x) = (x - 1)(x^2 + x - 6) \)

b) \( g(x) = (x + 2)^2(x - 3)^3(2x + 1)^4 \)

c) \( h(x) = x(x - 2)^5(x + 3)^2 \)

d) \( f(x) = 5x^3 - 4x^2 + 7x - 8 \)
Example 10: Referring to the graphs below:

i. Identify as even or odd degree polynomials

ii. Determine possible degrees and signs of leading coefficients

iii. Find possible zeros of the polynomials.

The zeroes of a polynomial \( y = p(x) \) and Multiplicity

Zeroes of a polynomial

Recall: If \( p(r) = 0 \) for a number \( r \), then \( r \) is called the zero of \( p \), to find the zeros of \( p \):

- Set \( p(x) = 0 \) and solve for \( x \).
- Factor, if it is possible to factor, the polynomial \( p \)

Example 11: Find the zeros of \( f(x) = x^3 + 2x^2 - 5x - 6 \).

Factor \( f(x) \) and set it equal to 0 and solve for \( x \).

\[
x^3 + 2x^2 - 5x - 6 = (x + 3)(x + 1)(x - 2)
\]

\( x = -3, -1, \text{ and } 2, \) are the zeros of the function.

Note that: \( f(-3) = 0, f(-1) = 0, \text{ and } f(2) = 0 \)

Multiplicity

Definition (Multiplicity)
The multiplicity of a zero is the number of times that zero occurs. For the polynomial function \( f(x) = (x - c)^k \), \( c \) is a zero of the function with multiplicity \( k \).

- If \( k \) is odd, then the graph crosses the x-axis at \((c, 0)\)
- If \( k \) is even, then the graph is tangent to the x-axis at \((c, 0)\) (touches the x-axis but does not cross it)
Theorem: Suppose \( P \) is a polynomial function and \( x = c \) is a zero of multiplicity \( m \). Then:

- If \( m \) is even, then the graph of \( P \) is **tangent to** the x-axis at \((c, 0)\) (touches and re-bounce from the x-axis at \((c, 0)\))
- If \( m \) is **odd**, then the graph of \( P \) **crosses** the x-axis at \((c, 0)\)

**Example 14:** For each of the following find the zeroes, state the multiplicity, and sketch the graph

a) \( f(x) = 5x(x - 2)^3(x + 1) \)

**Solution:** \( x = 2 \) is a zero with **multiplicity 3**; (graph crosses the at \( x = 2 \))
\( x = -1 \) is a zero with **multiplicity 1**; (graph crosses the at \( x = -1 \))
\( x = 0 \) is a zero with multiplicity 1; (graph crosses the at \( x = 0 \))

b) \( f(x) = -x^2(x - 1)^3(x + 2)^4 \).

**Solution:** \( x = -2 \) is a zero with **multiplicity 4**; (graph re-bounces at \( x = -2 \))
\( x = 1 \) is a zero with **multiplicity 3**; (graph crosses the at \( x = 1 \))
\( x = 0 \) is a zero with multiplicity 2; (graph re-bounces at \( x = 0 \))

**Example 3.1.6 page 245:** Reading

**Important ideas for sketching graphs of polynomials:**

- **Zeros and their multiplicity**
- **Degree and leading coefficient**
- **End Behavior:** The **leading term** and the **degree** tells us about the **end behavior**
- **Intercepts:** \( x \) and \( y \) - intercepts
- **Symmetries:** If any
- **Test Points:** Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros. Include the y-intercept on the table.
- **Graph:** Plot the intercept and other points you found on the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.
Example 12: Choose the correct graph of \( h(x) = -(x - 4)(x + 1)(x - 5) \)

![Graph options]

Example 13: Choose the correct graph of \( f(x) = x^5 + 3x^2 \)

![Graph options]

Example 15: Sketch the graph of the following polynomials

\[
\begin{align*}
\text{a)} & \quad f(x) = 6x^3 + 7x^2 - 3x + 1 \\
\text{b)} & \quad f(x) = (x - 1)(x^2 + x - 6) \\
\text{c)} & \quad g(x) = x(x + 2)(x - 3)^2(2x + 1) \\
\text{d)} & \quad h(x) = -(x - 1)(x - 3)(x + 2)(x + 1)
\end{align*}
\]

OER 1: West Texas A&M University Tutorial 35: Graphs of Polynomial Functions

OER 2: Notes and Exercises College of the Redwoods:

Chapter 6: Polynomial Functions

- Section 1: Polynomial Functions
  - Exercises with Answers
  - Exercises with Solutions
- Section 2: Zeros of Polynomials
  - Exercises with Answers
  - Exercises with Solutions
- Section 3: Extrema and Models
  - Exercises with Answers
  - Exercises with Solutions

Practice Problems from the Text
Page 246, Exercises 3.1.1: #1 – 32 (odd numbers)
Polynomial Divisions (Page 257)

Long Division and Synthetic Division

Long Division

The Division Algorithm

If $P(x)$ and $D(x)$ are polynomials, with $D(x) \neq 0$, then there are unique polynomials $Q(x)$ and $R(x)$, where $R(x)$ is either 0 or of degree less than the degree of $D(x)$, such that

$$P(x) = D(x) \cdot Q(x) + R(x)$$

The polynomials $P(x)$ and $D(x)$ are called the Dividend and divisor respectively $Q(x)$ is called the quotient $R(x)$ is called the remainder

For Example: If we divide $6x^2 - 26x + 12$ by $x - 4$ we get

$$6x^2 - 26x + 12 = (x - 4)(6x - 2) + 4$$

In the Division Algorithm Format:

$P(x) = 6x^2 - 26x + 12$ is the Dividend;
$D(x) = x - 4$ is the Divisor;
$Q(x) = 6x - 2$ is the Quotient and
$R(x) = 4$ is the Remainder

Example 1: Divide $(x^4 - 2x^2 + x - 2) \div (x^2 + x - 4)$

Solution: By Division Algorithm:

$$x^4 - 2x^2 + x - 2 = (x^2 + x - 4) \cdot Q(x) + R(x)$$

Where $Q(x)$ and $R(x)$ are polynomials to be determined using Polynomial long Division

In dividing polynomials using Long Division:

First we must insert zero placeholders for missing terms and rewrite the division as:

$$(x^4 + 0x^3 - 2x^2 + x - 2) \div (x^2 + x - 4)$$

Next, set up the polynomial division as a standard division problem and repeat the steps Divide, Multiply, Subtract, Carry Down over and over until the divisor no longer may be divided into the result at the bottom.
**Step 1:** We eliminate $x^4$ from the *dividend*, to do so we need to multiply the *divisor* by $x^2$ and subtract the product from the dividend and bring down $x$ to get a new dividend, $-x^3 + 2x^2 + x$.

**Step 2:** Next we eliminate $-x^3$ from the new dividend, to do so multiply the divisor by $-x$ and subtract the product from $-x^3 + 2x^2 + x$ and bring down $-2$, which gives a second new dividend $3x^2 - 3x - 2$. Repeat this process for the new dividend, until we get a dividend of degree smaller than the divisor, $x^2 + x - 4$.

\[
\begin{array}{c|ccc}
 & x^2 & x & -4 \\
\hline
 x^2 + x - 4 & x^4 & +0x^3 & -2x^2 + x & -2 \\
 & x^4 & +x^3 & -4x^2 & \\
\hline
0 & -x^3 & +2x^2 & +x & \\
 & -x^3 & +x^2 & +4x & \\
\hline
0 & +3x^2 & -3x & -2 & \\
 & 3x^2 & +3x & -12 & \\
 & & & -6x & +10 \\
\end{array}
\]

Since, $-6x + 10$ is of smaller degree than $x^2 + x - 4$, we stop the process here.

The polynomial $x^2 - x + 3$ is Quotient, and $-6x + 10$ is remainder; and so,

\[
x^4 - 2x^2 + x - 2 = (x^2 + x - 4) \cdot (x^2 - x + 3) + (-6x + 10)
\]

**Example 2:** Using Long Division, find the quotient and the remainder of each of the following.

a) \[ f(x) = \frac{3x^3 + 2x - 4}{x^2 - 4} \]

b) \[ f(x) = \frac{5x^4 + 3x^2 + 2x - 8}{2x^2 + 2x - 8} \]

c) \[ f(x) = \frac{x^4 + 2x^3 - 6x^2 - 2}{x - 1} \]

**OER:** West Texas A&M University Tutorial 36: Long Division

**Homework Practice Problems from the Text**

Exercise 3.2.1 Page 265 #1 – 6
**Synthetic Division - The Shortcut for Dividing by \((x - c)\)**

When dividing a polynomial \(f(x)\) by a linear factor \((x - c)\), we can use a shorthand notation saving steps and space.

**Procedure for Synthetic Division;** we proceed with example

**Example 3:** Divided \(f(x) = 3x^3 + 2x - 1\) by \((x - 4)\).

1. Insert zero placeholder for the missing term: \(f(x) = 3x^3 + 0x^2 + 2x - 1\)

2. Write the value of “c” and the coefficients of \(f(x)\) in a row. In Example 3 \(c = 4\), and the coefficients are 3, 0, 2, and -1.

   \[
   \begin{array}{cccc}
   4 & | & 3 & 0 & 2 & -1 \\
   \end{array}
   \]

3. Carry down the first coefficient. In this case carry down the 3.

   \[
   \begin{array}{cccc}
   4 & | & 3 & 0 & 2 & -1 \\
   \end{array}
   \]

   \[
   3
   \]

4. Multiply this carried down coefficient by the value of c. In this case, multiply \(3 \cdot 4 = 12\). Place this result in the next column.

   \[
   \begin{array}{cccc}
   4 & | & 3 & 0 & 2 & -1 \\
   \end{array}
   \]

   \[
   \begin{array}{cccc}
   3 & | & 3 & 12 & 1 \\
   \end{array}
   \]

5. Add the column entries and place result at bottom. In this case you add 0 + 12 to get 12. Multiply this addition result by “c” and place in next column. In this case you multiply \(12 \cdot 4 = 48\).

   \[
   \begin{array}{cccc}
   4 & | & 3 & 0 & 2 & -1 \\
   \end{array}
   \]

   \[
   \begin{array}{cccc}
   3 & | & 3 & 12 & 48 \\
   \end{array}
   \]

6. Repeat Step 4 for all columns. In this example, you get

   \[
   \begin{array}{cccc}
   4 & | & 3 & 0 & 2 & -1 \\
   \end{array}
   \]

   \[
   \begin{array}{cccc}
   3 & | & 3 & 12 & 48 & 200 \\
   \end{array}
   \]

7. The bottom row of numbers reveals the answer along with the remainder. In this case, the numbers 3, 12, 50, 199 indicate an answer of \(3x^2 + 12x + 50, \text{ with remainder } 199\) or \(3x^2 + 12x + 50 + 199/(x - 4)\)

**Note:** The answer will always have degree one less than the dividend.
Example 4: Using synthetic division, find the quotient and remainder

a) \( f(x) = \frac{x^4 + 2x^3 - 6x^2 - 2}{x - 1} \)

b) \( x^5 + 32 \div x + 2 \)

Example 3.2.1 Page 261: Reading

Homework Practice Problems from the Text
Exercise 3.2.1 Page 265 #7 – 20

The Factor Theorem:

For a polynomial \( f(x) \), if \( f(c) = 0 \), then \( x - c \) is a factor of \( f(x) \).

Note: \( x - c \) is a factor of \( f(x) \) means, the remainder when \( f(x) \) divided by \( x - c \) is 0

Example 5: Let \( f(x) = x^3 - 2x^2 \).

\( f(2) = 0 \), so by the Factor Theorem, \( x - 2 \) is a factor of \( f(x) \)

Example 6: Let \( f(x) = x^3 + 2x^2 - 5x - 6 \)

a) Use long division to determine whether \( x + 3 \) and \( x - 3 \) are factors of \( f(x) \).

b) Use The Factor Theorem to determine whether \( x + 3 \) and \( x - 3 \) are factors of \( f(x) \).

c) Use synthetic division to determine whether \( x + 3 \) and \( x - 3 \) are factors of \( f(x) \).

The Remainder Theorem:

If \( f(x) = (x - c)Q(x) + R \), then \( f(c) = R \). That is, the remainder when dividing the polynomial \( f(x) \) by \( x - c \) is the same as the value of the function evaluated at \( x = c \).

Example 7: Using the Remainder Theorem, find the remainder when \( f(x) = x^3 + 2x^2 - 5x - 6 \) is divided by:

a) \( x + 2 \)

b) \( x - 1 \)

Example 3.2.2 Page 362: Reading
Example 8: Decide whether the numbers $-3, \ 2,$ are zeros of the polynomial 
\[ f(x) = 3x^3 + 5x^2 - 6x + 18 \]; use both Synthetic Division and the Remainder Theorem

Example 9: Factor the polynomial $f(x) = x^3 + 5x^2 - 2x - 24$ and solve the equation $f(x) = 0$.

Solution:
1) First, list all integral factors of $-24$: which are $\pm 1, \ \pm 2, \ \pm 3, \ \pm 4, \ \pm 6, \ \pm 8, \ \pm 12, \ \pm 24$
2) Next, check if any of these factor is a zero of $f(x)$
   Check for: $f(\pm 1) = ?, \ f(\pm 2) = ?, \ etc.$
3) Finally using the result of 2) and division of polynomials factor $f(x)$

Example 10: Factor the polynomial $f(x) = x^4 + 2x^3 - 25x^2 - 50x$ completely

Example 11: Factor the polynomial $f(x) = x^4 - 8x^4 - 33$ completely

Example 12: Factor the polynomial $f(x) = x^4 - 5x^3 + 20x - 16$ completely

Example 12: Solve $x^3 + 4x^2 + 25x - 100 = 0$

OER: West Texas A&M University
   Tutorial 37: Synthetic Division and the Remainder and Factor Theorems

OER: West Texas A&M University on zeros of polynomial functions
   Tutorial 38: Zeros of Polynomial Functions, Part I
   Tutorial 39: Zeros of Polynomial Functions, Part II

Homework Practice Problems from the Text
   Exercise 3.2.1 Page 265 #21 – 46 (odd numbers)