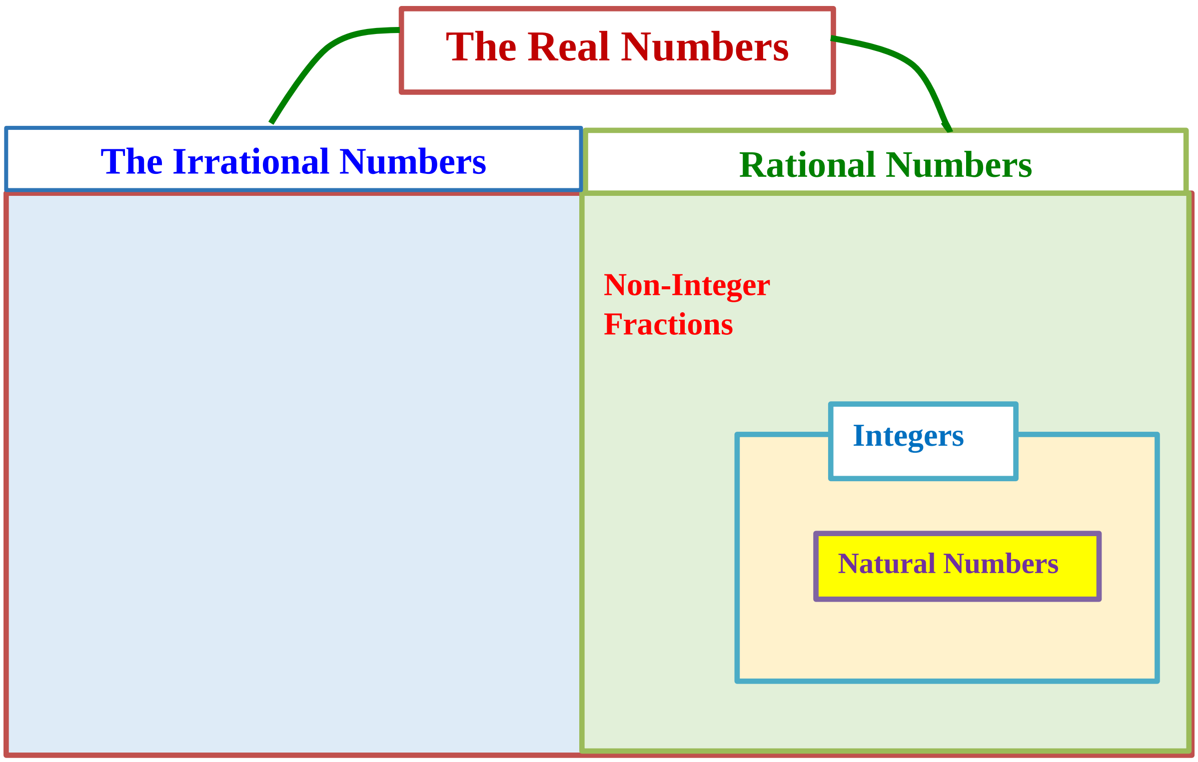
The Real Number System

The Set of Real Numbers **R** is made up **two** disjoint set of Numbers:

- The **Set of Rational Numbers** and
- The **Set of Irrational Numbers**

See diagram below



Example: YouTube videos:

- Rational and Irrational Numbers: https://www.youtube.com/watch?v=cLP7INqs3JM

Example: YouTube videos properties of real numbers:

- Distributive Property: https://www.youtube.com/watch?v=xC-fQ0KEzsM
- Commutative property: https://www.youtube.com/watch?v=UeG_EYd-0xw
- Associative property: https://www.youtube.com/watch?v=fUgkIcx82xY

Example: YouTube videos (PEMDAS)

- Order of operations 1: https://www.youtube.com/watch?v=ClYdw4d4OmA
- Order of operations 2: https://www.youtube.com/watch?v=piIcRV2dx7E
- Order of operations 3: https://www.youtube.com/watch?v=3Po3nfITsok

The Rational Numbers

Definition: (Rational Numbers)
A Rational Number is a number that **can** be written in the form $\boldsymbol{a/b}$; $\boldsymbol{a}$ and $\boldsymbol{b}$ **integers,** $\boldsymbol{b \neq 0}$**.**
In other words, a **Rational Number** is a number the **can be written** in a **fraction form**

Examples: Rational Numbers

a) -5, 11, 5/4, 22/7, 111/87, 0, -121, -1/3, 1/3, etc.

b) 0.333…, 5.33, -3.65, 0.242424…= $0.\overline{24}$, 3.612612612…= $3.\overline{612}$, etc.

Decimal Representation of a Rational Number

A Rational Number has a **decimal representation** that either **terminates** or **repeats**.

Example: **0.5** is a terminating decimal
0.333… $= \mathbf{0.\overline{3}}$ is a repeating decimal

Example: **Change** $\mathbf{2.\overline{7}}$ **in to a fraction.**

Solution: We use the following procedure for changing a repeating decimal in to a fraction.

Let $\boldsymbol{x = 2.\overline{7}}$. Since only one digit is repeating, we multiply both sides by 10 (If there were two digits repeating we multiply by 100, three digits repeating by a 1000 and so on) to get

$$\mathbf{10}\boldsymbol{x} = \mathbf{10} \times \mathbf{2.777} \ldots$$

$$\mathbf{10}\boldsymbol{x} = \mathbf{27.777} \ldots$$

Now we **take** the **difference** between $\mathbf{10}\boldsymbol{x}$ and $\boldsymbol{x}$

$$\mathbf{10}\boldsymbol{x} - \boldsymbol{x} = (\,\mathbf{27.777} \ldots) - (\mathbf{2.777} \ldots)$$

Note $\begin{array}{r} 27.777\ldots \\ -\ \underline{2.777\ldots} \\ 25.0 \end{array}$

Thus, $\mathbf{9}\boldsymbol{x} = \mathbf{25}$, then dividing both sides by **5** we get

$$\boldsymbol{x} = \frac{\mathbf{25}}{\mathbf{9}}$$

That is, $\boldsymbol{x} = \mathbf{2.\overline{7}} = \frac{\mathbf{25}}{\mathbf{9}}$

Example 1: Decimal Numbers

a) 23 = 23.0 Terminating decimal

b) 1.253 Terminating decimal

c) 1.333… Repeating Decimal

d) 3.612612612…= $3.\overline{612}$ Repeating Decimal

e) Any integer is a rational number

Example 2: Write the following numbers in fraction form

a) 1.33   d) $3.\overline{612}$

b) 1.333…   e) $0.\overline{12}$

c) -2.455

Example: YouTube Videos:

- Converting fractions to a decimals: https://www.youtube.com/watch?v=Gn2pdkvdbGQ
- Converting decimals to fractions: https://www.youtube.com/watch?v=DR2DYe7PI74
- Converting repeating decimals to fraction: https://www.youtube.com/watch?v=Ihws0d-WLzU

The Irrational Numbers

Definition: (Irrational Numbers)

An **Irrational Number** is a number that **cannot** be written in the form a/b; a and b **integers,** $b \neq 0$.

An **Irrational Number Cannot** be **written** in a **fraction form**

Example 3: Examples of Irrational numbers

a) 1.01001000100001…   **e)** $\sqrt{2}$

b) 0.12345…   **f) e**

c) **– 4.110111011110…**   **g)** $\sqrt[3]{7}$

d) π

Decimal Representation of an Irrational Number

An **Irrational Number** has a **decimal representation** that **neither terminates nor repeats**

Example 4:

a) $\sqrt{2} = 1.41421356237...$

b) – 4.110111011110…

c) $e = 2.71828182845...$

d) $\pi = 3.14159265358 ...$

Example 5: Show that $\sqrt{2}$ cannot be written as a fraction.

Proof: YouTube video

The square root of 2 is irrational: https://www.youtube.com/watch?v=mX91_3GQqLY

Important Notations of Set of Numbers

R – Denotes the set of **Real numbers**

Q – Denotes the set of **Rational numbers**

Z – Denotes the set of **Integers**

W– Denotes the set of **Whole numbers**

N – Denotes the set of **Natural numbers**

Summary Chart of the Number Systems