Chapter 2
Set Theory (page 42 -)

Objectives:

- Specify sets using both the listing and set builder notation
- Understand when sets are well defined
- Use the element symbol property
- Find the cardinal number of sets

Definition (sets)

In mathematical terms a collection of (**well defined**) objects is called a **set** and the **individual objects** in this collection are called the **elements** or **members** of the set.

Examples: a) **S** is the collection of all students in Math 1001 CRN 6977 class
b) **T** is the set of all students in Math 1001 CRN 6977 class who are 8 feet tall
d) **E** is the set of even natural numbers less than 2
e) **B** is the set of beautiful birds (Not a well-defined set)
f) **U** is the set of all tall people (Not a well-defined set)

Note: The sets in b) and d) have no elements in them.

Definition: (Empty Set): A set containing no element is called an **empty set** or a **null set.**

Notations: $\{\ \}$ ***or*** $\emptyset$ denotes empty set.

Representations of Sets

In general, we represent (describe) a set by **listing** it elements or by **describing** the property of the elements of the set, within curly braces.

Two Methods: Listing and **Set Builder**

Examples of Listing Method: List the elements of the set

a) $\mathbf{N_5}$ is the set of natural numbers less than 5
   $\mathbf{N_5} = \{\mathbf{1},\ \mathbf{2},\ \mathbf{3},\ \mathbf{4}\}$
b) $\mathbf{N_{100}}$ the set of positive integers less than 100
   $\mathbf{N_{100}} = \{\mathbf{1},\ \mathbf{2},\ \mathbf{3},\ .\ .\ .\ .\ ,\ \mathbf{100}\}$
c) $\boldsymbol{S} = \{\boldsymbol{m},\ +,\ \mathbf{1},\ \emptyset,\ \mathbf{5},\ \Delta\}$

Set Builder Method: has the general format $\{\boldsymbol{x}: \boldsymbol{P}(\boldsymbol{x})\}$, here $\boldsymbol{P}(\boldsymbol{x})$ is the **property** that the element $\boldsymbol{x}$ should **satisfy** to be in the collection

Examples, Set-builder Method:

d) $\boldsymbol{S} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ student\ in\ Math\ 1001 CRN\ 6977\ class\}$.
   Here $\boldsymbol{P}(\boldsymbol{x}) =$ $\boldsymbol{x}$ **is a student in Math 1001 CRN6977 class**
e) $R = \{x: x\ is\ a\ real\ number\ strictly\ between -1\ and\ 2\} = \{x: -1 < x < 2\}$
   Here $\boldsymbol{P}(\boldsymbol{x}) =$ $\boldsymbol{x}$ **is a real number strictly between -1 and 2**
f) $T = \{x: x\ is\ a\ solution\ of\ the\ equation\ x^4 - 1 = 0\} = \{x: x = -1,\ 1, -i,\ i\}$
   Here $\boldsymbol{P}(\boldsymbol{x}) =$ $\boldsymbol{x}$ satisfies $\mathbf{x^4 - 1 = 0}$

Example: YouTube Videos:

- Sets and set notation: https://www.youtube.com/watch?v=01OoCH-2UWc
- Set builder notation: https://www.youtube.com/watch?v=xnfUZ-NTsCE

Sets of Numbers commonly used in mathematics

$\boldsymbol{R} = \mathbb{R} = \{x: \boldsymbol{x\ is\ a\ real\ number}\} = \{x: \boldsymbol{x\ has\ a\ decimal\ expansion}\}$

$\boldsymbol{Q} = \mathbb{Q} = \{x: \boldsymbol{x\ is\ a\ rational\ number}\} = \left\{x: \boldsymbol{x}\ is\ of\ the\ form \frac{\boldsymbol{a}}{\boldsymbol{b}}\ where\ \ \boldsymbol{a}, \boldsymbol{b}\ \in \mathbb{Z}, \boldsymbol{b} \neq \boldsymbol{0}\right\}$

$\boldsymbol{I} = \boldsymbol{Z} = \mathbb{Z} = \{x: \boldsymbol{x\ is\ an\ integer}\} = \{\ .\ .\ .\ , \boldsymbol{-2}, \boldsymbol{-1},\ \boldsymbol{0},\ \boldsymbol{1},\ \boldsymbol{2},\ \ .\ .\ .\}$

$\boldsymbol{W} = \{x: \boldsymbol{x\ is\ a\ whole\ number}\} = \{\boldsymbol{0},\ \boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3},\ \ .\ .\ .\ \}$

$\boldsymbol{N} = \mathbb{N} = \{x: \boldsymbol{x\ is\ a\ natural\ number}\} = \{\boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3},\ \ .\ .\ .\ \}$

Definition (Universal Set)

The **universal set** is the **set of all elements under consideration** in a given discussion. We denote the universal set by the capital letter **U**.

The Element Symbol $(\in)$

We use the **symbol** $\in$ to stand for the phrase ***is an element of***, and the symbol $\notin$ means ***is not an element of***

Example1: i) $\boldsymbol{A} = \{\boldsymbol{0}, \boldsymbol{-3},\ \boldsymbol{10},\ \pm,\ \forall,\ \boldsymbol{\partial}, \boldsymbol{H}\ \}$,

$\boldsymbol{0} \in \boldsymbol{A}$, read as **0** is an element of A

$\boldsymbol{10}\ \in \boldsymbol{A}$, read as **10** is an element of A

$\boldsymbol{\partial} \in \boldsymbol{A}$**,** read as $\boldsymbol{\partial}$ is an element of A

$\boldsymbol{3} \notin \boldsymbol{A}$**,** read as **3 is not** an element of A

$\boldsymbol{h} \notin \boldsymbol{A}$**,** read as **h is not** an element of A

ii) $\boldsymbol{B} = \{\emptyset, \{\boldsymbol{0}, \emptyset\}, \{\emptyset\}, \{\boldsymbol{1}, \boldsymbol{0}\}, \boldsymbol{0}\}$. Referring to set **B** answer the following as **True** or **False**.

a) $\emptyset \in B$

b) $\{\emptyset\} \in B$

c) $\{0\} \in B$

d) $\{\{1, 0\}, 0\} \in B$

e) $\{0, \emptyset\} \in B$

f) $0 \notin B$

Cardinal Number

Definition: The **number** of elements in set **A** is called the **cardinal number** of **A** and is denoted by $\boldsymbol{n(A)}$. A set is **finite** if its **cardinal number** is a **whole number**. A set is **infinite** if it is **not finite**

Example 2: Find the cardinal number of

a) $\boldsymbol{A} = \{0, -3,\ 10,\ \pm,\ \forall,\ \partial, H\ \}$
b) $T = \{x: x\ is\ a\ solution\ of\ the\ equation\ x^4 - 1 = 0\}$
c) $B = \{\emptyset, \{0, \emptyset\}, \{\emptyset\}, \{1, 0\}, 0\}$
d) $E = \{\emptyset\}$
e) $H = \{\ \}$
f) $\mathbb{N} = \{\boldsymbol{1},\ \boldsymbol{2},\ \boldsymbol{3},\ \ .\ .\ .\ \}$

Example: YouTube Videos:

- Elements subsets and set equality: https://www.youtube.com/watch?v=kGyOrbbllEY&spfreload=10
- Introduction to Set Concepts & Venn Diagrams: https://www.youtube.com/watch?v=Jt-S9J947C8

2.2 Comparing Sets (Page 50)

Objectives:

- Determine when sets are equal
- Know the difference between the relations subsets and proper subsets
- Use Venn diagrams to illustrate sets relationships
- Distinguish between the ideas of "equal" and "equivalent" sets

Sets Equality

Definition (Equal sets)

Two sets **A** and **B** are **equal** if and only if they have **exactly** the **same members**. We write $\boldsymbol{A = B}$ to mean **A** is equal to **B.** If **A** and **B** are **not equal** we write $\boldsymbol{A \neq B}$**.**

Example 1: Let $\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ natural\ number\ Less\ than\ 4\}$

$\boldsymbol{B} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y}\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

$\boldsymbol{D} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ whole\ number\ less\ than\ 4\}$

$\boldsymbol{E} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

In the list above, identify the sets that are equal

Solution: $\boldsymbol{A} = \boldsymbol{B}$ and $\boldsymbol{D} = \boldsymbol{E}$

Subsets

Definition (subset):

The set **A** is said to be a **subset** of the set **B** if every **element** of **A** is also an **element** of **B**. We indicate this relationship by writing $\boldsymbol{A \subseteq B}$. If **A** is **not a subset** of **B**, then we write $\boldsymbol{A \nsubseteq B}$

Example 2: Let $\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ natural\ number\ Less\ than\ 4\}$

$\boldsymbol{B} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y}\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

$\boldsymbol{D} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ whole\ number\ less\ than\ or\ equal\ to\ 4\}$

In the set list above: $\boldsymbol{A} \subseteq \boldsymbol{B}$ and also $\boldsymbol{B} \subseteq \boldsymbol{A}$

$\boldsymbol{A} \subseteq \boldsymbol{C}$ but $\boldsymbol{C} \nsubseteq \boldsymbol{A}$**,** $\boldsymbol{B} \subseteq \boldsymbol{C} \subseteq \boldsymbol{D}$

Proper Subset

Definition (proper subset)**:**

A set **A** is said to be a **proper subset** of set **B** if $\boldsymbol{A}\ is\ a\ subset\ of\ \boldsymbol{B}$ but **B** is **not a subset** of **A.** We write $\boldsymbol{A \subset B}$ to mean **A** is a **proper subset** of **B.**

Example 3: Let $\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ natural\ number\ Less\ than\ 4\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y}\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 3\}$

$\boldsymbol{E} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$

Here $\boldsymbol{A} \subset \boldsymbol{C}$**, A** is a **proper subset** of **C**

$\boldsymbol{C} \subset \boldsymbol{E}$**, C** is a **proper subset** of **E**

Example: YouTube Videos:

- Subsets and proper subsets: https://www.youtube.com/watch?v=1wsF9GpGd00
- Subsets and proper subsets: https://www.youtube.com/watch?v=s8FGAclojcs

Example 4: **Let** $\boldsymbol{B} = \{\emptyset, \{\mathbf{0}, \emptyset\}, \{\emptyset\}, \{\mathbf{1}, \mathbf{0}\}, \mathbf{0}\}$. Referring to set **B,** answer the following as **True** or **False**.

a) $\emptyset \subseteq B$

b) $\{\emptyset\} \subseteq B$

c) $\{0\} \subseteq B$

d) $\{\{1,0\},0\} \subseteq B$

e) $\{0,\emptyset\} \subseteq B$

f) $0 \subseteq B$

Example 5: Finding **all subsets** of a set

Let $\boldsymbol{S} = \{\mathbf{0}, \mathbf{1}\}$, $\boldsymbol{T} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ and $\boldsymbol{E} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$**.**

List all **subsets** of the sets **S** and **T**

Solution:

$\boldsymbol{S} = \{\mathbf{0}, \mathbf{1}\}$**;** the subsets of set **S** are $\emptyset, \{\mathbf{0}\}, \{\mathbf{1}\}, \{\mathbf{0}, \mathbf{1}\}$. There are $4 = 2^2$ subsets of **S**

$\boldsymbol{T} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$; the subsets of set **T** are $\emptyset, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{c}\}, \{\boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$. There are $\mathbf{8} = \mathbf{2}^3$ Subsets of **T**

$\boldsymbol{E} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}\}$**, Set E** has $\mathbf{2^4} = \mathbf{16}$ subsets

Number of Subsets of a set:

If a set has **k** elements, then the **number of subsets** of the set is given by $\mathbf{2^{\boldsymbol{k}}}$**.**

Equivalent Sets

Definition:

Two sets **A** and **B** are **equivalent**, or in **one to one correspondence**, iff $\boldsymbol{n}(\boldsymbol{A}) = \boldsymbol{n}(\boldsymbol{B})$.
In other words, **two** sets are **equivalent** if and only if they have the **same Cardinality**.

Example 6: Equivalent sets

a) $\boldsymbol{T} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ and $\boldsymbol{B} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$are **equivalent, Note** $\boldsymbol{T} \neq \boldsymbol{B}$

b) $\mathbb{N} = \{\mathbf{1},\ \mathbf{2},\ \mathbf{3},\ ...\}$ and $\boldsymbol{W} = \{\mathbf{0},\ \mathbf{1},\ \mathbf{2},\ \mathbf{3},\ ...\}$ **are equivalent**

c) $\mathbb{N} = \{\mathbf{1},\ \mathbf{2},\ \mathbf{3},\ ...\}$ and $\mathbb{Z} = \{\boldsymbol{x}: \boldsymbol{x}\ \boldsymbol{is\ an\ integer}\}$ **are equivalent**

d) $\mathbb{N} = \{\mathbf{1},\ \mathbf{2},\ \mathbf{3},\ ...\}$ and $\mathbb{Q} = \{\boldsymbol{x}: \boldsymbol{x}\ \boldsymbol{is\ a\ rational\ number}\}$ **are equivalent**

e) **Equal** sets are **equivalent**

Example 7: Let $A = \{0,\ a,\ e,\ \pi,\ \{0\}\}$. How many subsets of A have:

a) One element (list the subsets)
b) Two elements (list the subsets)
c) Four elements (list the subsets)

Set Operations (page 57)

Objectives:

- Perform the set operations of union, intersection, complement and difference
- Understand the order in which to perform set operations
- Know how to apply DeMorgan's Laws in set theory
- Use Venn diagrams to prove or disprove set theory statements
- Use the Inclusion – Exclusion Principle to calculate the cardinal number of the union of two sets

Union of Sets

Definition (set union ∪):

The union of two sets **A** and **B**, written $\boldsymbol{A \cup B}$, is the set of elements that are members of **A** or **B** (or both). Using the **set-builder notation**, $\boldsymbol{A \cup B = \{x: x \in A \text{ or } x \in B\}}$

The union of more than two sets is the set of all elements belonging to at least to one of the sets.

Example 1:

$\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x} \; is\ a\ natural\ number\ Less\ than\ 7\ and\ greater\ than\ 1\}$

$\boldsymbol{B} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y} \; is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

$\boldsymbol{D} = \{\boldsymbol{x}: \boldsymbol{x} \; is\ a\ whole\ number\ less\ than\ 4\}$

Find a) $\boldsymbol{A \cup B}$ c) $\boldsymbol{A \cup B \cup D}$

b) $\boldsymbol{A \cup C}$ **d)** $\boldsymbol{C \cup B \cup D}$

Intersection of Sets

Definition (set intersection ∩)

The intersection of two sets **A** and **B**, written $\boldsymbol{A \cap B}$, is the set of elements common to both **A** and **B**.

Using the **set-builder notation**, $\boldsymbol{A \cap B = \{x: x \in A \text{ and } x \in B\}}$

The intersection of more than two sets is the set of all elements that belongs to each of the sets. If the intersection, $\boldsymbol{A \cap B = \emptyset}$**,** then we say **A** and **B** are **disjoint**.

Example 2:

$\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x} \; is\ a\ natural\ number\ Less\ than\ 7\ and\ greater\ than\ 2\}$

$\boldsymbol{B} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y} \; is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

$\boldsymbol{D} = \{\boldsymbol{x}: \boldsymbol{x} \; is\ a\ whole\ number\ less\ than\ 3\}$

Find a) $\boldsymbol{A \cap B}$ c) $\boldsymbol{A \cap B \cap C}$

b) $\boldsymbol{A \cup D}$ d) $\boldsymbol{C \cap B \cap D}$

Example 3: Let $\boldsymbol{B} = \{\emptyset, \{\boldsymbol{0}, \emptyset\}, \{\emptyset\}, \{\boldsymbol{1}, \boldsymbol{0}\}, \boldsymbol{0}\}$ and $\boldsymbol{A} = \{\{\emptyset, \boldsymbol{0}\}, \{\boldsymbol{0}\}, \{\emptyset\}, \{\boldsymbol{0}, \boldsymbol{1}\}, \boldsymbol{1}\}$. Find:

a) $\boldsymbol{n(B)}$ c) $\boldsymbol{A \cap B}$

b) $\boldsymbol{n(A)}$ d) $n(A \cap B)$

Example: YouTube Videos:

- Intersection and union of sets: https://www.youtube.com/watch?v=jAfNg3ylZAI

Set Complement

Definition (A', $A\ prime\ or\ A\ complement$)

If **A** is a subset of the universal set **U**, the **complement** of **A** is the set of **elements** of **U** that are **not elements** of **A**. This set is denoted by A'**.** Using the set-builder notation, $A' = \{x: x \in U\ but\ x \notin A\}$

Example 4: Using Venn diagram:

a) Show that, if $A \subseteq B$, then $A \cap B = A$ b) Show that, if $A \subseteq B$, then $A \cup B = B$

Example 5: Let $U = \{0, 1, 2, 3, . . . , 10\}$ and

$A = \{x: x\ is\ a\ natural\ number\ Less\ than\ 7\ and\ greater\ than\ 2\}$

$B = \{1, 2, 3\}$ $C = \{y: y\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

$D = \{x: x\ is\ a\ whole\ number\ less\ than\ 3\}$

Find a) A' **b)** B' c) C' d) D'

e) $A' \cup B'$ f) $C' \cup B'$ g) $(C \cup D)'$ h) $(B \cap C)'$

Set Difference

Definition (**B – A**, **B less A**)

The **difference** of sets **B and A** is the set of **elements** that are **in B** but **not in A**. This set is **denoted** by $B - A$. Using the set-builder notation, $B - A = \{x: x \in B\ and\ x \notin A\}$

Example 6: Let $A = \{x: x\ is\ a\ natural\ number\ Less\ than\ 7\ and\ greater\ than\ 2\}$

$B = \{1, 2, 3\}$ $C = \{y: y\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\}$

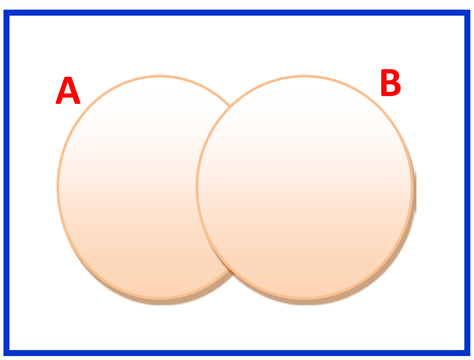
$D = \{x: x\ is\ a\ whole\ number\ less\ than\ 3\}$

Find: a) $A - B$ b) $D - C$ c) $C - B$ d) $A - (B - C)$

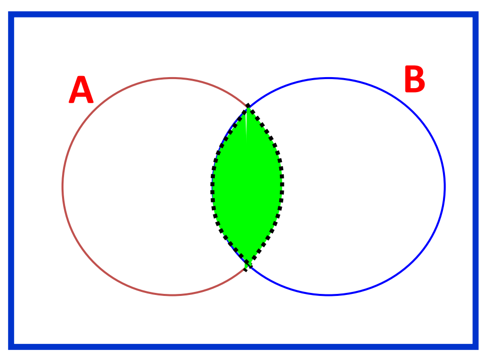
Venn Diagrams

The Universal Set (**U**) is **represented** by a **rectangle.** The **shaded regions represent**, respectively, the **union**, **intersection, difference** and **complement** of the sets A and B.

a) $A \cup B$

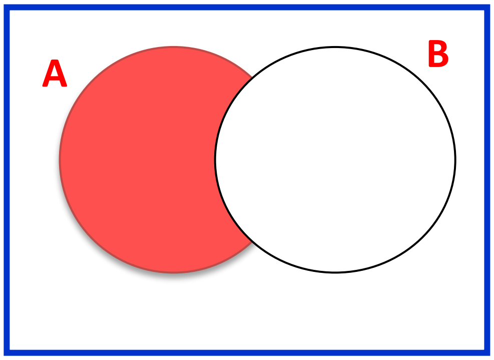


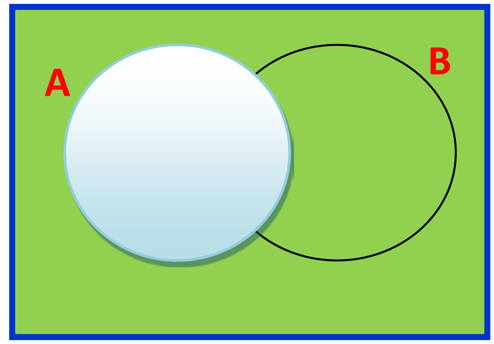
b) $A \cap B$



Example: YouTube Videos:

- Intersection and union of sets: https://www.youtube.com/watch?v=jAfNg3ylZAI

c) $A - B$ 

d) A' 

Example: YouTube Videos:

- Difference or relative complement: https://www.youtube.com/watch?v=2B4EBvVvf9w
- Absolute complement: https://www.youtube.com/watch?v=GVZUpOm3XUg

Order of Set Operations

Just as we perform **arithmetic operations** in a certain order, set notations specifies the order in which we perform set operations.

1. Just like with numbers, we always do anything in **parentheses first**. If there is more than one set of **parentheses**, we work from the **inside out**.

2. **Union**, **intersection**, and **difference** operations are **all equal** in the order of **precedence**. So, if we have more than one of these at a time, we have to use **parentheses** to indicate which of these operations should be done first.

Properties of Set Operations: If **A, B** and **C** be sets, then

a) $A \cup B = B \cup A$ **Commutative** property of **union**

b) $A \cap B = B \cap A$ **Commutative** property of **intersection**

c) $(A \cup B) \cup C = A \cup (B \cup C)$ **Associative** property of **union**

d) $(A \cap B) \cap C = A \cap (B \cap C)$ **Associative** property of **intersection**

e) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ **Distributive** property of **union** over **intersection**

f) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ **Distributive** property of **intersection** over **union**

Proof: Use Venn diagram

Note that: $A - B - C$, $A \cup B \cap C$, $A - B \cup C$, $A \cap B \cup C$, $A \cap B - C$ and so on, without **parenthesis** indicating which to do first, are **ambiguous**.

Example 7: Let $U = \{0, 1, 2, 3, . . . \; , \; 10\}$ be the universal set and

$A = \{x: x \text{ is a natural number Less than } 9 \text{ and greater than } 2\} = \{3, 4, 5, 6, 7, 8\}$

$B = \{1, 2, 3\}$ $\quad C = \{y: y \text{ is a positive integer less than or equal to } 4\} = \{1, 2, 3, 4\}$

$D = \{x: x \text{ is a whole number less than } 3\}$

Find a) $(A - B) - C$ b) $A - (B - C)$ c) $A \cap (B \cup C)$ j) $(A \cup B)'$

d) $(A \cap B) \cup C$ e) $(A \cap B) - C$ f) $A \cap (B - C)$

g) $A' \cap B'$ h) $(A \cap B)'$ i) $A' \cup B'$

DeMorgan's Laws:

If **A** and **B** are sets, then a) $(A \cup B)' = A' \cap B'$ and

b) $(A \cap B)' = A' \cup B'$

The inclusion-Exclusion Principle

If **A** and **B** are sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example 8: Let **U** $= \{0, 1, 2, 3, 4, 5, 6, a, b, c, d, e, f, g\}$, the universal set, $A = \{0, 1, 2, 3, 4\}$, $B = \{a, b, 1, 2, 3, c, e\}$, $C = \{0, b, g, d\}$, $D = \{0, 1, 3, e, 5, 6\}$, and $E = \{0, 1, 6, a, f\}$.

Find: a) $A \cap \left(A' \cap B'\right)$ b) $(A \cup B) \cap (D - C)$ c) $(E \cap C)' \cap (D \cup B)$

d) Verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

f) **Verify DeMorgan's Laws** for the sets **B** and **D**

Example: YouTube Videos:

- Bringing the set operations together: https://www.youtube.com/watch?v=OCNXS_m1HWU

Solving Survey Problems with Venn Diagrams

Objectives:

- Label sets in Venn diagrams with various names
- Use Venn diagrams to solve survey problems
- Understand how to handle contradictory information in survey problems

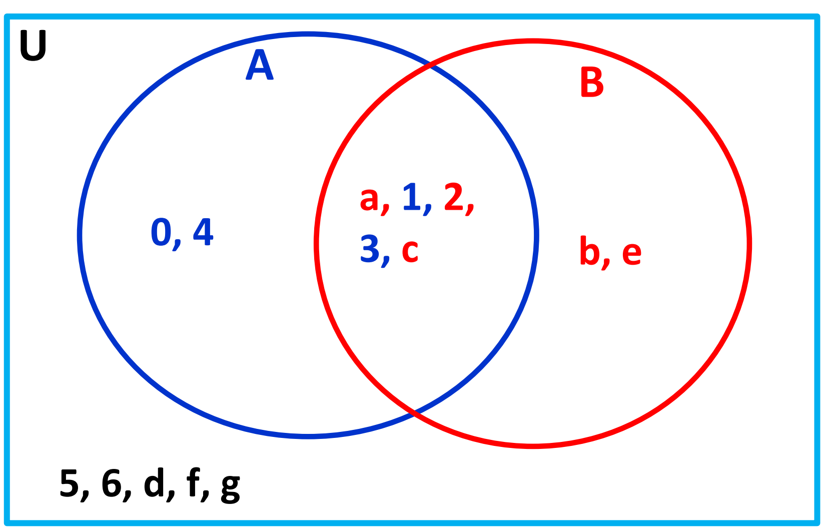
Example: YouTube Videos

- Solving problems using Venn Diagrams: https://www.youtube.com/watch?v=MassxXy8iko

Example 1: Describe the following sets using Venn diagram

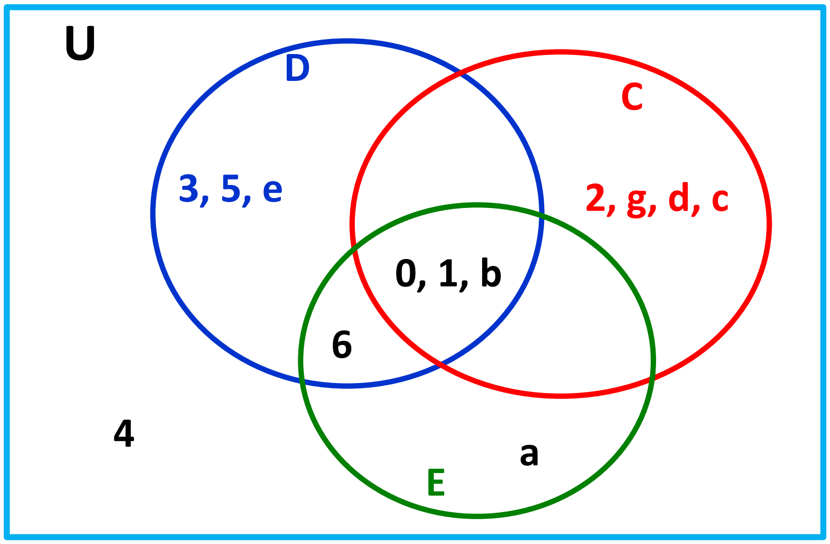
a) Let **U** $= \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}\}$ be the universal set $A = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \boldsymbol{a}, \boldsymbol{c}\}$ and $B = \{\boldsymbol{a}, \boldsymbol{b}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \boldsymbol{c}, \boldsymbol{e}\}$.

Solutions:



b) Let $\mathbf{U} = \{\ \mathbf{0},\ \mathbf{1},\ \mathbf{2},\ \mathbf{3},\ \mathbf{4},\ \mathbf{5},\ \mathbf{6},\ \boldsymbol{a},\ \boldsymbol{b},\ \boldsymbol{c},\ \boldsymbol{d},\ \boldsymbol{e},\ \boldsymbol{f},\ \boldsymbol{g}\ \}$, the universal set

$\boldsymbol{C} = \{\mathbf{0},\ \mathbf{1},\ \mathbf{2},\ \boldsymbol{b},\ \boldsymbol{g},\ \boldsymbol{d},\ \boldsymbol{c}\ \}$, $\boldsymbol{D} = \{\mathbf{0},\ \mathbf{1},\ \mathbf{3},\ \boldsymbol{e},\ \boldsymbol{b},\ \mathbf{5},\ \mathbf{6}\ \}$, and $\boldsymbol{E} = \{\mathbf{0},\ \mathbf{1},\ \mathbf{6},\ \boldsymbol{a},\ \boldsymbol{b},\ \boldsymbol{f}\ \}$



Example 2: Represent Using Venn diagram. Let $\mathbf{U} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \mathbf{10}\}$ be the universal set and

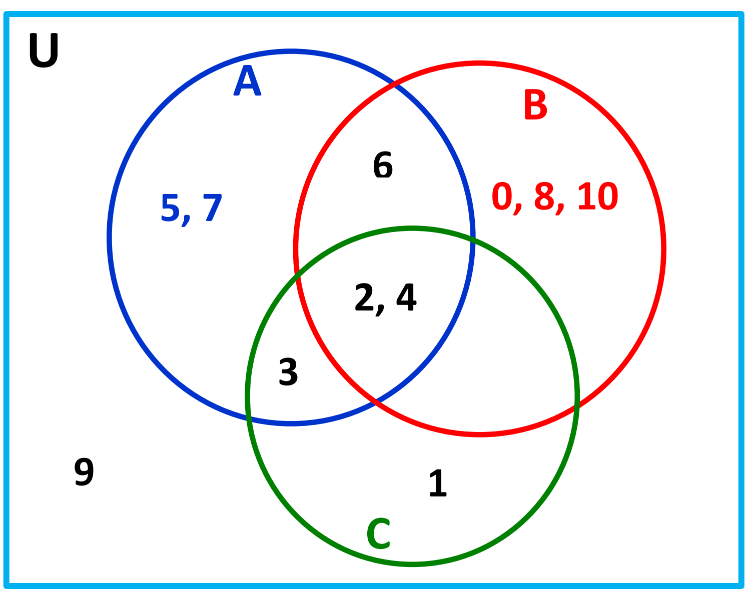
$\boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ a\ natural\ number\ Less\ than\ 8\ and\ greater\ than\ 1\} = \{\mathbf{2},\ \mathbf{3},\ \mathbf{4},\ \mathbf{5},\ \mathbf{6},\ \mathbf{7}\}$

$\boldsymbol{B} = \{\boldsymbol{x}: \boldsymbol{x}\ is\ an\ even\ whole\ number\ less\ than\ or\ equal\ to\ \mathbf{10}\} = \{\mathbf{0}, \mathbf{2}, \mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{10}\}$

$\boldsymbol{C} = \{\boldsymbol{y}: \boldsymbol{y}\ is\ a\ positive\ integer\ less\ than\ or\ equal\ to\ 4\} = \{\ \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$

Solution:

$\mathbf{U} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \mathbf{10}\}$, $\boldsymbol{A} = \{\mathbf{2},\ \mathbf{3},\ \mathbf{4},\ \mathbf{5},\ \mathbf{6},\ \mathbf{7}\}$, $\boldsymbol{B} = \{\mathbf{0}, \mathbf{2}, \mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{10}\}$, $\boldsymbol{C} = \{\ \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$



Example 3: For each of the following, using the given information, find the number of elements in set A, B, and C.

a) $A \cap B = \{\ \}, n(A \cap C) = 5, n(B \cap C) = 3, n(C - A) = 7, n(A - C) = 2, n(U) = 14$

b) $n(A \cap B) = 5, n(A \cap B \cap C) = 2,\ n(C \cap B) = 6,\ n(B - A) = 10, n(B \cup C) = 23,$

$n(A \cap C) = 7, n(A \cap B \cup C) = 31$

Solution: Use Venn diagram

Survey Problems

Example 4: The **N**ational **R**esource **D**efense **C**ouncil, which was instrumental in drafting California's Global Warming Solution Act, believes that by using technology properly, we can cut U.S. global warming solution by half. Three of the solutions proposed by **NRDC** are using **energy-efficient appliances**, driving **energy-efficient cars**, and using **renewable energy sources**. Assume that you surveyed 100 members of Congress to determine which solutions they favored funding and obtained the following results:

a) 12 favored the increased use of **renewable** energy source **only**
b) 20 recommended funding **both** energy-efficient **appliances** and **renewable** energy sources
c) 22 favored funding **both** energy-efficient **cars** and increased use of **renewable** energy sources
d) 14 want to fund **all three** areas

From this information, determine the **total number** who favored increased funding for **renewable energy**.

Solution: **Good Notations**; we will represent this information using sets

$$EA = \boldsymbol{A} = \{\boldsymbol{x}: \boldsymbol{x}\ \boldsymbol{favors\ energy\ efficient - appliances}\}$$

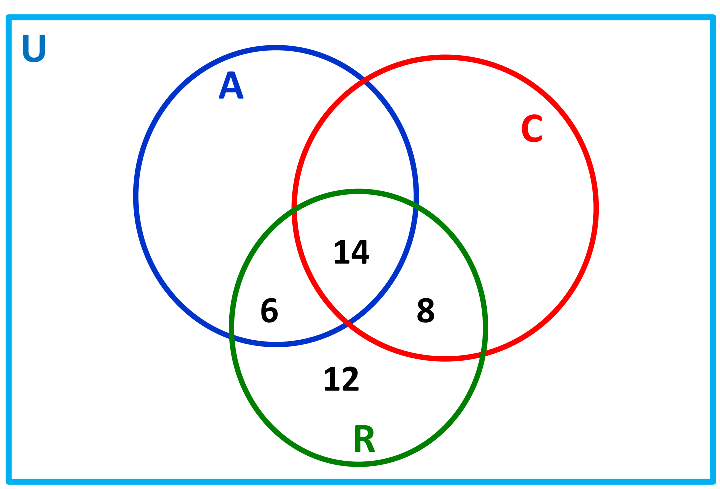
$$EC = \boldsymbol{C} = \{\boldsymbol{x}: \boldsymbol{x}\ \boldsymbol{favors\ energy\ efficient - cars}\}$$

$$RE = \boldsymbol{R} = \{\boldsymbol{x}: \boldsymbol{x}\ \boldsymbol{favors\ renuable\ energy\ sources}\}$$

The conditions from **a) – d)** can be written using set notations as follows:

$$\boldsymbol{n(R \cap A' \cap C') = 12}, \quad \boldsymbol{n(A \cap R) = 20}, \quad \boldsymbol{n(C \cap R) = 22}, \quad \boldsymbol{n(A \cap C \cap R) = 14}$$

Venn diagram representations of the sets:



The total number who favored renewable energy is $\mathbf{6+14+8+12=40}$

Example 5: Fitness Survey

Personal Fitness Magazine surveyed a group of young adults regarding their exercise programs and the following results are obtained:

a) 3 were using **resistance training**, **Tae Bo**, **and Pilates** to improve their fitness.
b) 5were using **resistance training and Tae Bo**
c) 12 were using **Tae Bo and Pilates**
d) 8 were using **resistance training and Pilates**
e) 15 were using **resistance training only**
f) 30 were using **Tae Bo**
g) 17 were using **Pilates** but **not Tae Bo**.
h) 14 were using **something other than** these **three** types of workouts

1) How many people were surveyed?
2) How many people were using only Tae Bo?
3) How many people were using Pilates but not resistance training?

Solution: Using set Notations we will represent this information using sets

The group of young adults surveyed is the universal set **U** containing **three subsets**.

$P=\{x: x \text{ is a young adults using Pilates}\}$

$T=\{x: x \text{ is a young adult using Tae Bo}\}$

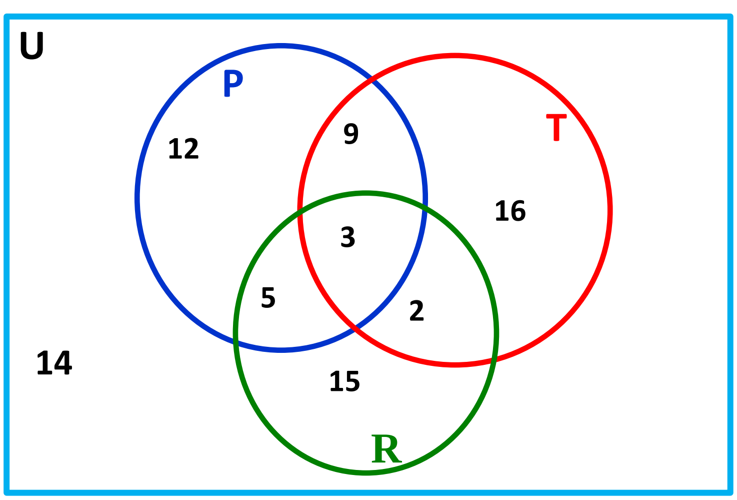
$R=\{x: x \text{ is a young adult using Resistance Training}\}$

The conditions from a) – h) can be written using set notations as follows:

$n(R\cap T\cap P)=3$ $\quad n(R\cap T)=5$ $\quad n(T\cap P)=12$ $\quad n(R\cap P)=8,$

$n(R\cap P'\cap T')=15,$ $\quad n(T)=36,$ $\quad n(P\cap T')=17,$ $\quad n(P'\cap T'\cap R')=14,$

Venn diagram representation of the sets:



1) How many people were surveyed? $\mathbf{14+12+15+16+19=76}$
2) How many people were using only Tae Bo? **16**
3) How many people were using Pilates but not resistance training? $\mathbf{9+12=21}$

Example 6: Social Media Survey

In a survey of 100 college students, the following information is obtained regarding their use of several social media sites:

a) 21 use Facebook, LinkedIn, and Twitter
b) 32 use both LinkedIn and Twitter
c) 44 use Facebook and Twitter
d) 31 use Facebook but not LinkedIn
e) 78 use Facebook or Twitter
f) 12 use only Twitter
g) 5 use none of these three sites

How many of those surveyed use Facebook?
How many do not use Twitter?

Solution: Using set Notations; we will represent this information using sets
The 100 group of students surveyed is the universal set **U** containing **three subsets**.

$\boldsymbol{F = \{x: x \text{ is a student using Facebook}\}}$

$\boldsymbol{L = \{x: x \text{ is a students using LinkedIn}\}}$

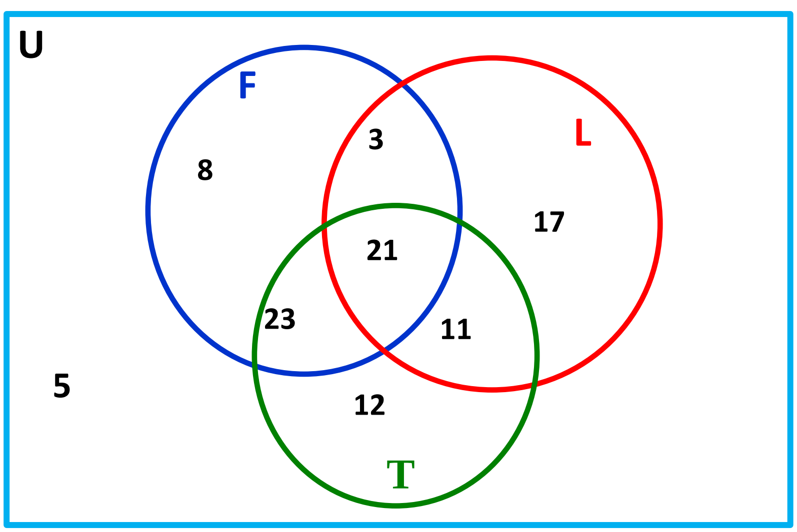
$\boldsymbol{T = \{x: x \text{ is a students using Twitter}\}}$

The conditions from a) – g) can be written using set notations as follows:

$\boldsymbol{n(F \cap L \cap T) = 21}$ $\boldsymbol{n(L \cap T) = 32}$ $\boldsymbol{n(F \cap T) = 44}$ $\boldsymbol{n(F \cap L') = 31,}$

$\boldsymbol{n(F \cup T) = 78,}$ $\boldsymbol{n(T \cap L' \cap F') = 12,}$ $\boldsymbol{n(F' \cap T' \cap L') = 5}$

Venn diagram representation of the sets:



How many of those surveyed use Facebook? $\boldsymbol{8 + 3 + 21 + 23 = 55}$
How many do not use Twitter? $\boldsymbol{8 + 3 + 17 + 5 = 33}$

Example 7: Survey of TV Preferences (Example 4 Page 70)

A television survey conducted a market survey to determine the **evening viewing** preferences of people in the 18 – 25 age bracket. The following information was obtained:

a) 3 prefer a **reality show early** on **weekdays**
b) 14 want to watch **TV early** on **weekdays**
c) 21 want to see **reality** shows **early**
d) 8 want **reality** shows on **weekdays**
e) 31 want to watch **TV** on **weekdays**
f) 36 want to watch **TV early**
g) 40 want to see **reality shows**
h) 13 prefer **late**, **weekends** shows that are **not reality shows**

How many people **do not want** to see **reality shows?**

How many prefer to **watch TV** on the **weekend**.

Solution: Using set Notations; we will represent this information using sets

The set of people surveyed is a universal set containing **three subsets**.

$W = \{x: x \text{ wants to watch TV on weekdays}\}$

$E = \{x: x \text{ wants to watch early TV shows}\}$

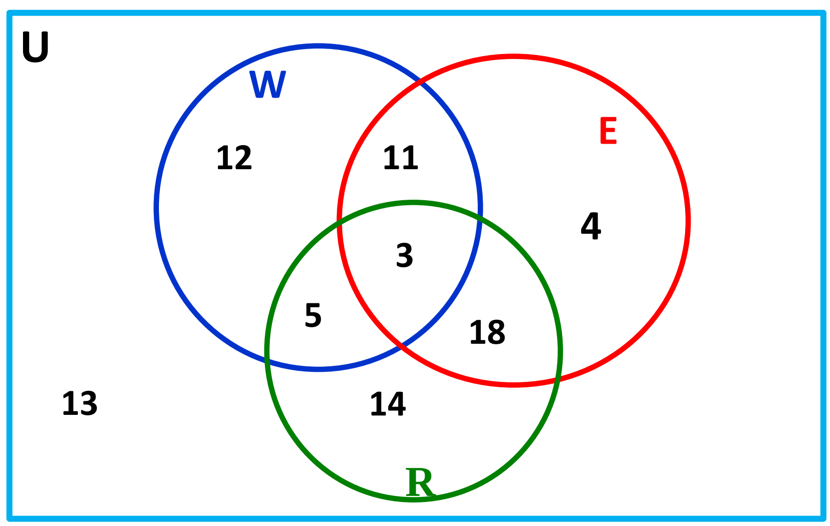
$R = \{x: x \text{ wants to watch reality TV shows}\}$

The conditions from **a) – h)** can be written using set notations as follows:

$$n(W' \cap E' \cap R') = 13, \quad n(R) = 40, \quad n(E) = 36, \quad n(W) = 31, \quad n(R \cap W) = 8,$$

$$n(R \cap E) = 21, \quad n(E \cap W) = 14, \quad n(R \cap W \cap E) = 3$$

Venn diagram representation of the sets:



Number of people who do not want to see the reality show is $12 + 11 + 4 + 13 = 40$

Number of people who watch the TV on weekends is $4 + 18 + 14 + 13 = 49$

Example 8: online music survey

Pandora.com surveyed a group of subscribers regarding which online music channels they use on a regular basis. The following information summarizes their answers:

a) 7 listened to **rap**, **heavy metal** and **alternative rock**
b) 10 listened to rap and heavy metal
c) 13 listened to heavy metal and alternative rock
d) 12 listened to rap and alternative rock
e) 17 listened to rap
f) 24 listened to heavy metal
g) 22 listened to alternative rock

1) How many people were surveyed?
2) How many people listened to either rap or alternative rock?
3) How many listened to heavy metal only?

Example 9: Academic Services Survey

The Dean of Academic Services surveyed a group of students about which support services they were using to help them improve their academic performance and found the following results.

a) 5 were using **office hours**, **tutoring**, and **online study groups** to improve their grades
b) 16 were using office hours and tutoring
c) 28 were using tutoring
d) 14 were using tutoring and online study groups
e) 8 were using office hours and online study groups but not tutoring
f) 23 were using office hours but not tutoring
g) 18 were using only online study groups
h) 37 were using none of these services

1) How many students were surveyed?
2) How many students were using only office hours?
3) How many students were using online study groups?

Example 9: News Sources Survey

A survey of young adults was taken to determine which of the various sources they use to obtain news. Of the 36 people who use the internet, 13 use the internet only to learn the news. Of the 48 people who use the newspaper, 11 use the newspaper only for the news coverage. There are 23 people who use newspapers, the internet and the television for the news coverage. From this information, determine how many use both the Internet and television to learn the news.