Chapter 1 – The Dawn of Quantum Theory

* By the Late 1800's
  - Chemists had
    -- generated a method for determining atomic masses
    -- generated the periodic table based on empirical observations
    -- resolved the structure of benzene
    -- elucidated the fundamentals of chemical reactions
  - Physicists had
    -- generated the relationship between heat and work
    -- developed the first two laws of thermodynamics
    -- demonstrated the wavelike nature of light
    -- applied statistical mechanics to chemical systems

* *Sounds great so what's the problem?*
  - The general scientific community believed:
    -- atoms are the basic constituents of matter
    -- Newton's Laws were universal
    -- all the phenomenon in the world is deterministic
  - There were several experiments which could not be explained based on this dogma and here are a few of them:
    -- black body radiation
    -- the photoelectric effect
    -- discrete atomic spectra
  - What conclusions do these experiments lead to?
    -- atoms are not the smallest/most microscopic object
    -- we need something beside Newtonian physics to explain these experiments

* And then came quantum mechanics …
  - explains these unsolved issues
  - explains bonding, structure and reactivity
  - uses probability instead of determinism
  - generates rules for electrons in atoms and molecules

* Let's talk about these persnickety experiments
  - Black Body Radiation
    What is it?
    -- Objects when heated will turn from red to white to blue which is an increase in energy/frequency
    -- the exact frequency emitted is dependent upon the composition of the body
    -- an ideal body absorbs/emits all frequencies and hence is also called a blackbody and the radiation that is emitted blackbody radiation
    Classical Physics Breakdown
    -- classical physics assumed this emission of light was a result of oscillating e-'s which act as antennae and can oscillate equally well at any frequency, ν
    -- Rayleigh-Jeans Law: used classical physics to generate the relationship between

spectral density, ρ(ν, T), and ν

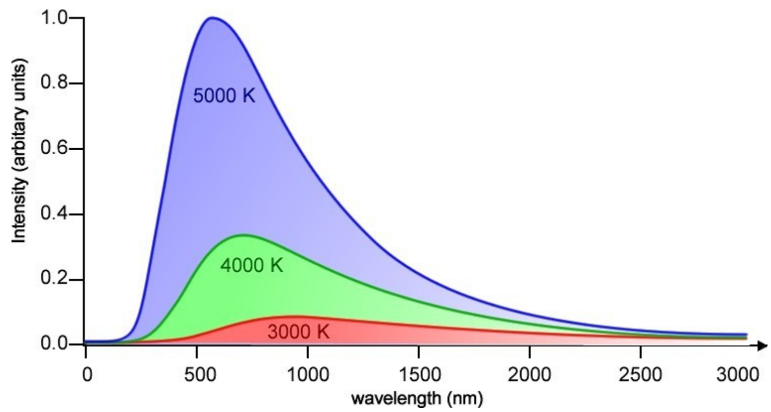
$$d\rho(\nu,T) = \rho_\nu(T)d\nu = \frac{8\pi k_B T}{c^3}\nu^2 d\nu \rightarrow \boxed{\rho_\nu(T) \propto \nu^2}$$

where $\rho_\nu(T)d\nu$ is the radiant energy density btwn ν and $\nu + d\nu$

$$k_B = R/N_A = \frac{8.314 \frac{J}{mol \bullet K}}{6.022\times10^{23} \frac{particles}{mol}} = 1.380\times10^{-23} \tfrac{J}{K} \ (\text{Boltzmann constant})$$

$$T = \text{absolute temperature (K)} \quad c = 2.998\times10^{8} \tfrac{m}{s} \ (\text{speed of light})$$

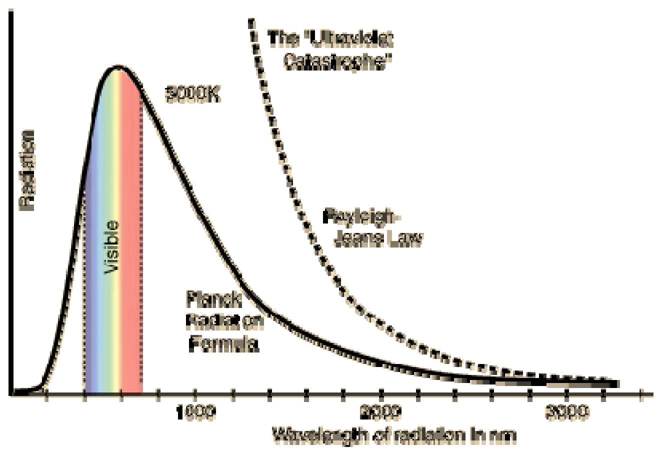
--- Experimentally we should see the graphs below: Similar to Figure 1.1 from the text



http://www.topmarksed.com/blog/2015/06/05/a-comprehensive-guide-to-black-body-radiation-in-the-hsc/

---- As T is increased you can see the color change

--- Unfortunately, we actually see a break down with Rayleigh-Jeans Law called the UV-Vis catastrophe where as energy increased/wavelength reduced the RJL goes to infinity rather than back to zero



http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html#c4

--- the dashed line is ν^2 and is consistent with the Rayleigh-Jeans Law at low T

--- this relationship does not work at high temperatures – called the **UV catastrophe** --- **<u>classical physics failure!</u>**

So, how do we fix this? Planck to save the day

--- Planck proposed the energy of these oscillating electrons ∝ frequency or

$E = nh\nu$ where $n = 1, 2, \ldots$ and h is proportionality constant

---- PLOT TWIST: Planck was one of the first to recognize variables may not have a continuum of values but instead be quantized

---- Blackbody radiation according to Planck

$$d\rho(\nu, T) = \rho_\nu(T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT} - 1}d\nu \rightarrow \boxed{\rho_\nu(T) \propto \nu^3}$$

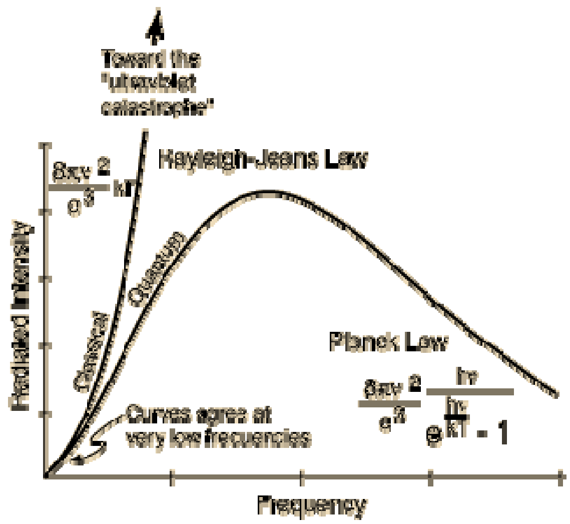
----- This expression can reproduce the RJLaw at low frequencies or for hν << k_BT

Recall the Taylor Series for $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$ for $-\infty < x < \infty$

$$\therefore\ e^{h\nu/k_BT} - 1 = 1 + \frac{h\nu}{k_BT} + \left(\frac{h\nu}{k_BT}\right)^2 \frac{1}{2!} + \ldots - 1$$

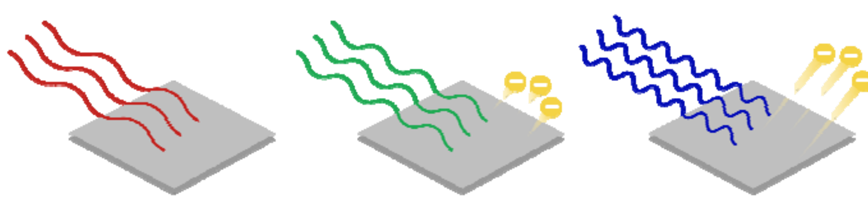
as $h\nu \rightarrow 0$ $\quad e^{h\nu/k_BT} - 1 = 1 + \frac{h\nu}{k_BT} + \left(\frac{h\nu}{k_BT}\right)^2 \frac{1}{2!} + \ldots - 1 \sim \frac{h\nu}{k_BT}$

$$\rho_\nu(T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/k_BT} - 1}d\nu = \frac{8\pi \cancel{h}}{c^3}\frac{\nu^{\cancel{3}2}k_BT}{\cancel{h}\cancel{\nu}}d\nu = \boxed{\frac{8\pi k_BT}{c^3}\nu^2 d\nu} \leftarrow RJLaw$$



http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html#c4

- Photoelectric Effect
  -- definition: electrons are emitted from a metallic surface when exposed to UV radiation



https://www.khanacademy.org/science/physics/quantum-physics/photons/a/photoelectric-effect

-- Classical physics states

that light is an electric field, $\vec{E}$, oscillating ⊥ to its direction of propagation and the intensity of the radiation $\propto \vec{E}^2$

--- the e-'s should oscillate along with the field and as the intensity increases so should these oscillations which will eventually lead to the ejection of an e- from the surface of the metal – **WRONG!**

--- the photoelectric should occur for any frequency as long as the intensity of the incident radiation is sufficiently high – **WRONG!**

-- Experimentally:

--- the kinetic energy of the ejected e- is independent of the intensity of the incident radiation

--- there is a threshold frequency, ν_0, which is dependent upon the metal

---- below this threshold no e-'s will be ejected from the surface

---- above this threshold the K.E. of the e-'s varies linearly with ν

-- Einstein to the rescue, he proposed:

--- light is made up of energy packets aka photons aka quanta

--- the energy of a photon is proportional to the light frequency, $E = h\nu$

$$E = h\nu \rightarrow K.E. = \frac{1}{2} m\nu^2 = h\nu - \Phi$$

--- Φ is called the work function and is analogous to the ionization energy of an isolated metallic atom (remember we are taking away an e-)

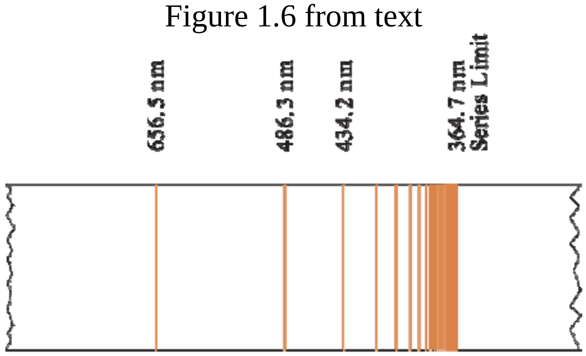
--- since $\frac{1}{2} m\nu^2$ must be ≥ 0, then $h\nu \geq \Phi$ or $h\nu_0 = \Phi$ hence $K.E. = h\nu - h\nu_0$

--- the constant h that Einstein predicted matched that of Planck's – **SUCCESS!**

- Hydrogen Atom Spectrum

-- In the 19th century scientists knew that each atom possessed a characteristic emission spectrum

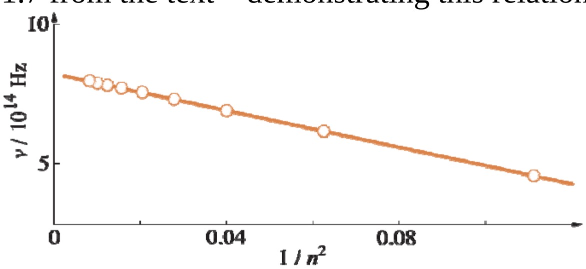
Figure 1.6 from text



--- these are called line spectra since they emit energy at a select number of frequencies – once again the spectrum is **not continuous** but **discrete -- quantized**

-- Balmer was the first one to show that these line spectra followed a particular pattern, $\nu \propto n^{-2}$ where n = 3, 4, 5, …

Figure 1.7 from the text – demonstrating this relationship



--- From this pattern he derived the relationship:

$$\nu = 8.2202\times10^{14}\left(1-\frac{4}{n^2}\right)Hz \text{ and } \tilde{\nu} = 109680\left(\frac{1}{2^2}-\frac{1}{n^2}\right)cm^{-1} \quad n = 3,4,5\ldots$$

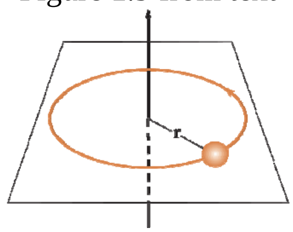
--- This relationship will give rise to all of the visible emissions for H, but what about the rest? Here comes Rydberg

-- Rydberg develops a formula which includes all of the possible emission lines of hydrogen

$$\tilde{\nu} = \frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)cm^{-1} \text{ where } n_2 > n_1$$

* Angular Momentum – Spinning Right Round

Figure 1.9 from text



- If we rotate a single particle around a fixed point with radius r we can write the kinetic energy T as:

$$T = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mr^2\omega^2 = \tfrac{1}{2}I\omega^2 \quad \text{where I is the moment of inertia}$$

- When we write the K.E. in terms of momentum: $linear: T = \frac{p^2}{2m} \quad angular: T = \frac{l^2}{2I}$

* Bohr, the hydrogen atom and Rydberg
  - the hydrogen atom
    -- consists of a massive positive nucleus and a smaller negative e- which is in a fixed orbit about the centrally located nucleus
    -- A tale of two forces
      --- Coulomb's law: force of attraction btwn an e- and a proton (the nucleus of hydrogen)

$$f = \frac{e^2}{4\pi\varepsilon_0 r^2}$$ where -e is the charge of an e-, e the charge of a proton, r is the radius

and ε_0 is the permittivity $\approx 8.854\times10^{-12}\, C^2/J\bullet m$

--- Centrifugal force: $f = \frac{m_e v^2}{r}$ where m_e is the mass of an e-

--- these two must be equal in order to ensure the e- doesn't speed toward nucleus

---- Solving for r we will reproduce the Bohr radius of hydrogen:

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

Bohr's orbit - forcing angular momentum to be quantized:

$$m_e vr = n\frac{h}{2\pi} = n\hbar \rightarrow v = \frac{n\hbar}{m_e r}$$

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow \frac{m_e}{r}\left(\frac{n\hbar}{m_e r}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow \frac{\cancel{m_e} n^2\hbar^2}{m_e^{\cancel{2}} r^{\cancel{3}}} = \frac{e^2}{4\pi\varepsilon_0 \cancel{r^2}}$$

$$\rightarrow r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2} = \frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2}$$

for n = 1, r = 52.92 pm the Bohr radius

- Bohr's Assumptions:
  - -- there are stable atomic states in which atoms do not radiate
    - --- these states are given by E_n with n = 1, 2, 3, … where n = 1 is the lowest energy state or ground state and is the most negative
  - -- angular momentum is quantized or these stationary orbits require an integer number of de Broglie wavelengths
- Total E of our e-:

  P.E. for an e- and a proton separated by distance r is $-\frac{e^2}{4\pi\varepsilon_0 r}$

$$E = K.E. + P.E. = T + V = \tfrac{1}{2} mv^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

recall $\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \rightarrow \tfrac{1}{2} m_e v^2 = \frac{e^2}{8\pi\varepsilon_0 r}$

$$\therefore E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

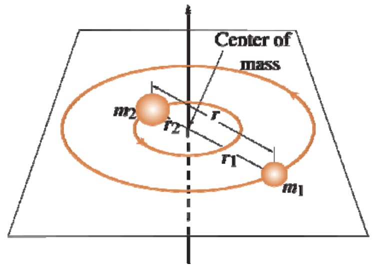
substituting $r = \frac{\varepsilon_0 n^2 h^2}{\pi m_e e^2}$ yields $E_n = -\frac{e^2}{8\pi\varepsilon_0} \bullet \frac{\pi m_e e^2}{\varepsilon_0 h^2 n^2} = -\frac{m_e e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2}$ *where* $n = 1, 2, 3, \ldots$

- Relationship btwn E_n and Rydberg
  - -- $\Delta E_n = E_f - E_i = h\nu = -\frac{m_e e^4}{8\varepsilon_0^2 h^2}\frac{1}{n_f^2} + \frac{m_e e^4}{8\varepsilon_0^2 h^2}\frac{1}{n_i^2} = \frac{m_e e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$
  - -- This expression looks suspiciously like the Rydberg expression

$$h\nu = hc\tilde{\nu} \text{ or } \tilde{\nu} = \frac{m_e e^4}{8\varepsilon_0^2 ch^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \therefore \frac{m_e e^4}{8\varepsilon_0^2 ch^2} \sim R_H$$

- Meet Reduced Mass

Figure 1.11 from text



-- at the center of mass $m_1 r_1 = m_2 r_2$

-- as we have said previously: $T = \frac{1}{2} I \omega$

-- where our moment of inertia can be written in terms of reduced mass:

$$reduced\ mass\ \mu = \frac{m_1 m_2}{m_1 + m_2} \ \&\ I = \mu r^2$$

-- looking back at our H-atom the reduced mass turns out to be m_e

-- Overall, the Bohr model works great for any H-like system (He^+ or Li^{2+})

- Limitations of this lovely description
  -- does not work for a system containing more than one e-
  -- fails when a magnetic field is applied to the system

* Wave-particle duality – here comes de Broglie

- classical optics supports the idea of light as a wave, e.g. refraction, etc.
- the photoelectric effect suggests that it can also be thought of as a particle
- enter de Broglie: he proposed that if light which is clearly a wave can act as particle than why can't a particle act as a wave
- Einstein proved that wavelength, λ, and momentum, p, are inversely proportional:

$$\lambda = \frac{h}{p}$$

- de Broglie claimed matter would also follow this relationship
  -- for matter p = mv where m = mass and v = velocity
  -- therefore the de Broglie wavelength is given by $\lambda = \frac{h}{mv}$
  -- but if matter acts like a wave then why aren't we all oscillating?
- Example: What is the de Broglie wavelength of 75 kg boy and an electron each traveling at 10 mph?

$$\lambda_{boy} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\, J \bullet s}{75 kg \times 10 \frac{miles}{hour}} = \frac{6.626 \times 10^{-34} \frac{kg \bullet m^2}{s}}{75 kg \times 10 \frac{miles}{hour} \times \frac{1.6093 km}{miles} \times \frac{hour}{3600 s} \times \frac{1000 m}{km}} = 1.98 \times 10^{-36} m$$

Too small to be detectable

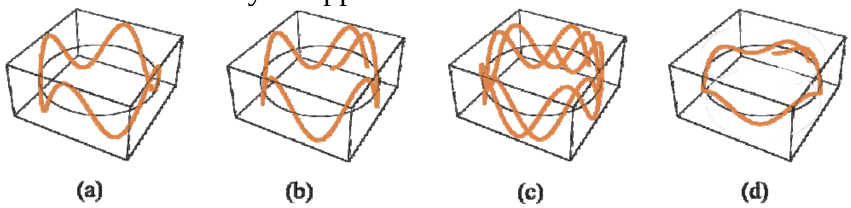
$$\lambda_e = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\, J \bullet s}{9.109 \times 10^{-31} kg \times 10 \frac{miles}{hour}} = \frac{6.626 \times 10^{-34} \frac{kg \bullet m^2}{s}}{9.109 \times 10^{-31} kg \times 10 \frac{miles}{hour} \times \frac{1.6093 km}{miles} \times \frac{hour}{3600 s} \times \frac{1000 m}{km}} = 1.63 \times 10^{-7} m$$

On the order of UV

* de Broglie Applied to Bohr's H-atom Model

- Figure 1.12 text: (a) represents the Bohr assumption and (b) –

(d) show what happens if the integer assumption is not in place – the wave will eventually disappear

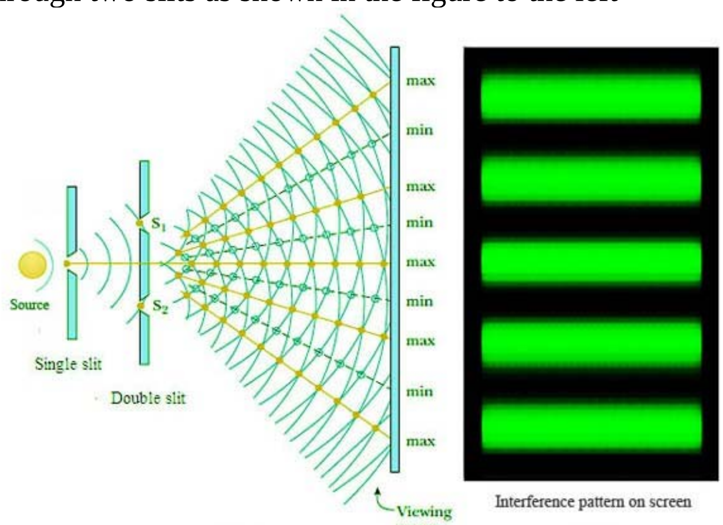


$2\pi r = n\lambda \text{ where } n = 1, 2, 3, \ldots$ since $\lambda = \frac{h}{p}$, then $2\pi r = n\frac{h}{p} = \frac{nh}{m_e v}$

$v = \frac{nh}{2\pi m_e r} = \frac{n\hbar}{m_e r} \text{ where } \hbar = h/2\pi$

* de Broglie In Real Life
- X-ray Diffraction
  -- occurs when X-rays are fired at a crystalline substance and is due to the interatomic spacing being on the order of the X-rays
  -- this phenomenon is another example of wave-particle duality
- Electron Microscopes
  -- Uses applied voltage through an electromagnetic field and are able to generate a much sharper images than their forefathers

* Two-Slit Experiments
- a light wave is initially allowed to pass through one slit and either hit or pass through two slits as shown in the figure to the left



https://practicallawandjustice.liberty.me/the-double-slit-experiment-a-rational-explanation/

- what results in a pattern of bands in which the light spaces are where the wave have acted in a constructive way and the dark is where they have acted destructively as shown on the right
- Below is a video of what happens when we allow only one particle at time to pass in our experiment:
  https://www.youtube.com/watch?v=TT-_uCLwKhQ

* More Uncertainty - Heisenberg
  - Heisenberg uncertainty principle: the exact momentum and the position of e- cannot be know simultaneously or $\Delta x \Delta p \geq h$
    - -- if we wish to know the location of an e- within a certain distance Δx we need a light source whose resolution is on the order of Δx or $\Delta x \approx \lambda$
    - -- unfortunately as soon as we shine this light on our e- we change its momentum, Δp
    - -- using the de Broglie relationship we obtain the Heisenberg uncertainty principle
    - -- we will be revisiting this later
  - Consequences of this uncertainty
    - -- we do not know what the velocity is if we know the e- is in the atom
    - -- Bohr assumed that the e- was a particle with known velocity and position
    - -- in order to complete the picture we need a true wavelike description of e-'s